## Study of the $K^{-} \rightarrow \mu^{-} \bar{\nu} \pi^{0}$ decay

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#### Abstract

The decay $K^{-} \rightarrow \mu^{-} \bar{\nu} \pi^{0}$ has been studied using in-flight decays detected with "ISTRA+" setup operating in the 25 GeV negative secondary beam of the U-70 PS. About 112 K events were used for the analysis. The $\lambda_{+}$and $\lambda_{0}$ slope parameters of the decay formfactors $f_{+}(t), f_{0}(t)$ have been measured : $\lambda_{+}=0.0321 \pm 0.004$ (stat) $\pm 0.002$ (syst) $\lambda_{0}=0.0209 \pm 0.004$ (stat) $\pm 0.002$ (syst); the correlation $d \lambda_{0} / d \lambda_{+}=-0.46$ The limits on the possible tensor and scalar couplings have been derived: $f_{T} / f_{+}(0)=-0.021 \pm 0.028$ (stat) $\pm 0.014$ (theory); $f_{S} / f_{+}(0)=0.004 \pm 0.005$ (stat) $\pm 0.005$ (theory)




Figure 1: The layout of the "ISTRA+" setup.

## 1 Introduction

The decay $K \rightarrow \mu \nu \pi^{0}\left(K_{\mu 3}\right)$ is known to be a key one in hunting for phenomena beyond the Standard Model (SM). In particular, significant efforts have been invested into T-violation searches by the measurements of the muon transverse polarization $\sigma_{T}$. In our analysis, based on $\sim 112 \mathrm{~K}$ events of the decay, we present new search for S and T interactions by fitting the $K_{\mu 3}$ Dalitz plot distribution, similar to as it was done for the $K_{e 3}$ decay: [1]. Another subject of our study is the measurement of the $\mathrm{V}-\mathrm{A} f_{+}(t) f_{0}(t)$ formfactor slopes $\lambda_{+}$and $\lambda_{0}$.

## 2 Experimental setup

The experiment is performed at the IHEP 70 GeV proton synchrotron U-70. The experimental setup "ISTRA+" has been described in some details in our recent paper on $K_{e 3}$ decay [回]. A schematic view of the detector is shown in Fig.1. The setup is located in the 4A negative unseparated secondary beam. The beam momentum is $\sim 25 \mathrm{GeV}$ with $\Delta p / p \sim 2 \%$. The admixture of $K^{-}$in the beam is $\sim 3 \%$. The beam intensity is $\sim 3 \cdot 10^{6}$ per 1.9 sec. U-70 spill.

## 3 Event selection

During 3 weeks physics run in March-April 2001, when the muon identification was in full operation, 363M events were logged on DLT's. This information is supported by about 100 M MC events generated with Geant3 [2]. Some information on the reconstruction procedure is presented in [1], here we touch only points relevant for the $K_{\mu 3}$ events selection.

The muon identification is based on the information from the $S P_{1}$ - a 576 -cell lead glass calorimeter and HC- a scintillator-iron sampling hadron calorimeter, subdivided into 7 longitudinal sections $7 \times 7$ cells each [3]. The calorimeters are located at the very end of the setup, after the main magnet (M2) and the last elements of the tracking system: drift tubes (DT) and the matrix scintillation hodoscope (MH). The first requirement is that the energy of the $S P_{1}$ cluster, associated with the charged track is less than $\sim 2.5$ MIP's; the HC energy, associated with the track should also be less than 2.5 MIP's. The last selection requires that more than $10 \%$ of the HC associated energy is deposited in 2 last layers (out of 7 ) of the HC. The efficiency


Figure 2: The $\gamma \gamma$ mass spectrum for the events with the identified muon and two extra showers.
of the algorithm to muons is tested on $K \rightarrow \mu \nu$ events and is found to be $\sim 70 \%$. The $\pi \rightarrow \mu$ misindification is measured on $K^{-} \rightarrow \pi^{-} \pi^{0}$ decay and is $\sim 3 \%$. After the muon identification, the selection of the events with two extra showers results in $M_{\gamma \gamma}$ spectrum shown in Fig.2. The $\pi^{0}$ peak has a mass of $M_{\pi 0}=134.6 \mathrm{MeV}$, and a resolution of 8.6 MeV . The missing mass squared- $\left(P_{K}-P_{\mu}-P_{\pi^{0}}\right)^{2}$, where P are the corresponding four-momenta, is presented in Fig.3. The cut is $\pm 0.01 \mathrm{GeV}^{2}$. The further selection is done by the requirement that the event passes $2 \mathrm{C} K \rightarrow \mu \nu \pi^{0}$ fit. The missing energy $E_{K}-E_{\mu}-E_{\pi^{0}}$ after this selection is shown in Fig. 4 The peak at low $E_{\text {miss }}$ corresponds to the remaining $K^{-} \rightarrow \pi^{-} \pi^{0}$ background. The corresponding cut is $E_{\text {miss }}>1.4 \mathrm{GeV}$. The surviving background is estimated from MC to be less than $4 \%$. The detailed data reduction information is shown in Table.1.

## 4 Analysis

The event selection described in the previous section results in selected 112K events in 2001 data. The distribution of the events over the Dalitz plot is shown in Fig.5. The variables $y=2 E_{\mu} / M_{K}$ and $z=2 E_{\pi} / M_{K}$, where $E_{\mu}, E_{\pi}$ are the energies of the muon and $\pi^{0}$ in the kaon c.m.s are used. The most general Lorentz invariant form of the matrix element for the decay $K \rightarrow \mu \nu \pi^{0}$ is 斗:

$$
\begin{equation*}
M=\frac{G_{F} \sin \theta_{C}}{\sqrt{2}} \bar{u}\left(p_{\nu}\right)\left(1+\gamma^{5}\right)\left[m_{K} f_{S}-\frac{1}{2}\left[\left(P_{K}+P_{\pi}\right)_{\alpha} f_{+}+\left(P_{K}-P_{\pi}\right)_{\alpha} f_{-}\right] \gamma^{\alpha}+i \frac{f_{T}}{m_{K}} \sigma_{\alpha \beta} P_{K}^{\alpha} P_{\pi}^{\beta}\right] v\left(p_{\mu}\right) \tag{1}
\end{equation*}
$$



Figure 3: The missing four-momentum squared $\left(P_{K}-P_{\mu}-P_{\pi^{0}}\right)^{2}$ for the selected events . The points with errors are the data, the histogram- MC.


Figure 4: The missing energy for the $\mu \pi^{0}$ events. The points with errors are the data, the histograms- MC. The dark(blue) peak at zero value corresponds to the MC-predicted $K \rightarrow \pi^{-} \pi^{0}$ background. The arrow indicates the cut value.

Table 1: Event reduction statistics.

| Run | March-April 2001 |
| :---: | :---: |
| $N_{\text {events }}$ on tapes | 363.002 .105 |
| Beam track reconstructed | $268.564 .958=74 \%$ |
| One secondary track found | $134.227 .095=37 \%$ |
| Written to DST | $107.215 .783=30 \%$ |
| $\mu^{-}$identified and $\pi^{0}$ identified | 218.813 |
| $\left\|M_{\text {miss }}^{2}\right\|<0.01$ | 195.799 |
| $K \rightarrow \mu \nu \pi^{0}$ accepted | 166.495 |
| $E_{\text {miss }}>1.4 \mathrm{GeV}$ | 112.157 |

It consists of scalar, vector and tensor terms. $f_{S}, f_{T}, f_{ \pm}$are functions of $t=\left(P_{K}-P_{\pi}\right)^{2}$. In the Standard Model (SM) the W-boson exchange leads to the pure vector term. The "induced" scalar and/or tensor terms, due to EW radiative corrections are negligibly small, i.e the nonzero scalar/tensor form factors indicate a physics beyond SM.

The term in the vector part, proportional to $f_{-}$is reduced(using the Dirac equation) to a scalar formfactor. In the same way, the tensor term is reduced to a mixture of a scalar and a vector formfactors. The redefined $f_{+}(\mathrm{V}), F_{S}(\mathrm{~S})$ and the corresponding Dalitz plot density in the kaon rest frame $\rho\left(E_{\pi}, E_{\mu}\right)$ are [5]:

$$
\begin{align*}
V & =f_{+}+\left(m_{\mu} / m_{K}\right) f_{T} \\
S & =f_{S}+\left(m_{\mu} / 2 m_{K}\right) f_{-}+\left(1+\frac{m_{\mu}^{2}}{2 m_{K}^{2}}-\frac{2 E_{\mu}}{m_{K}}-\frac{E_{\pi}}{m_{K}}\right) f_{T} \\
\rho\left(E_{\pi}, E_{\mu}\right) & \sim A \cdot|V|^{2}+B \cdot \operatorname{Re}\left(V^{*} S\right)+C \cdot|S|^{2}  \tag{2}\\
A & =m_{K}\left(2 E_{\mu} E_{\nu}-m_{K} \Delta E_{\pi}\right)-m_{\mu}^{2}\left(E_{\nu}-\frac{1}{4} \Delta E_{\pi}\right) \\
B & =m_{\mu} m_{K}\left(2 E_{\nu}-\Delta E_{\pi}\right) \\
C & =m_{K}^{2} \Delta E_{\pi} ; \Delta E_{\pi}=E_{\pi}^{\max }-E_{\pi} ; E_{\pi}^{\max }=\frac{m_{K}^{2}-m_{\mu}^{2}+m_{\pi}^{2}}{2 m_{K}}
\end{align*}
$$

Following [6] a scalar formfactor $f_{0}$ is introduced: $f_{0}(t)=f_{+}(t)+\frac{t}{m_{K}^{2}-m_{\pi}^{2}} f_{-}(t)$ and linear dependence of $f_{+}, f_{0}$ on t is assumed: $f_{+}(t)=f_{+}(0)\left(1+\lambda_{+} t / m_{\pi}^{2}\right) ; f_{0}(t)=f_{+}(0)\left(1+\lambda_{0} t / m_{\pi}^{2}\right)$. Then $f_{-}=f_{+}(0)\left(\lambda_{0}-\lambda_{+}\right) \frac{m_{K}^{2}-m_{\pi}^{2}}{m_{\pi}^{2}}$.

The procedure for the experimental extraction of the parameters $\lambda_{+}, \lambda_{0}, f_{S}, f_{T}$ starts from the subtraction of the MC estimated background from the Dalitz plot of Fig.4. The background


Figure 5: Dalitz plot ( $y=2 E_{\mu} / M_{K} ; z=2 E_{\pi^{0}} / M_{K}$ ) for the selected $K \rightarrow \mu \nu \pi^{0}$ events after the 2 -C fit.
normalization was determined by the ratio of the real and generated $K^{-} \rightarrow \pi^{-} \pi^{0}$ events. Then the Dalitz plot was subdivided into $20 \times 20$ cells. The background subtracted distribution of the numbers of events in the cells ( $\mathrm{i}, \mathrm{j}$ ) over the Dalitz plot, for example, in the case of simultaneous extraction of $\lambda_{+}, \lambda_{0}$ and $\frac{f_{S}}{f_{+}(0)}$, was fitted with the function:

$$
\begin{equation*}
\rho(i, j) \sim \sum_{k_{i} ; k_{1}+k_{2}+k_{3}=0,1,2} W_{k_{1} k_{2} k_{3}}(i, j) \cdot \lambda_{+}^{k_{1}} \cdot \lambda_{0}^{k_{2}} \cdot\left(f_{S} / f_{+}(0)\right)^{k_{3}} \tag{3}
\end{equation*}
$$

Here $W_{k_{1} k_{2} k_{3}}$ are MC-generated functions, which are build up as follows: the MC events are generated with constant density over the Dalitz plot and reconstructed with the same program as for the real events. Each event carries the weight w determined by the corresponding term in formula 2, calculated using the MC-generated("true") values for y and z. The radiative corrections according to [7] were taken into account. Then $W_{k_{1} k_{2} k_{3}}$ is constructed by summing up the weights w of the events in the corresponding Dalitz plot cell. This procedure allows to avoid the systematic errors due to the "migration" of the events over the Dalitz plot because of the finite experimental resolution.

## 5 Results

The results of the fit are summarized in Table.2.
The first line corresponds to pure V-A SM fit. The first column is independent fit of our $K_{\mu 3}$ data. The $\lambda_{+} \div \lambda_{0}$ correlation parameter is: $\frac{d \lambda_{0}}{d \lambda_{+}}=-0.46$. The $\lambda_{+}$value $\lambda_{+}^{\mu}=0.0321 \pm 0.004$

Table 2: Results of the fit.

|  | $\mu^{-} \bar{\nu} \pi^{0}$ | $\mu^{-} \bar{\nu} \pi^{0}+e^{-} \bar{\nu} \pi^{0}$ |
| :---: | :---: | :---: |
| $\lambda_{+}$ | $0.0321_{-0.0040}^{+0.0040}$ | $0.0296_{-0.0014}^{+0.0014}$ |
| $\lambda_{0}$ | $0.0197_{-0.0047}^{+0.046}$ | $0.0209_{-0.0042}^{+0.0042}$ |
| $\lambda_{+}$ | $0.0321_{-0.0040}^{+0.0040}$ | $0.0297_{-0.0014}^{+0.0014}$ |
| $\lambda_{0}$ | 0.01700 | 0.01700 |
| $f_{S} / f_{+}(0)$ | $0.0034_{-0.0058}^{+0.0058}$ | $0.0039_{-0.0052}^{+0.0052}$ |
| $\lambda_{+}$ | $0.0338_{-0.0037}^{+0.037}$ | $0.0299_{-0.0014}^{+0.0014}$ |
| $\lambda_{0}$ | 0.01700 | 0.01700 |
| $f_{T} / f_{+}(0)$ | $-0.0240_{-0.0326}^{+0.0330}$ | $-0.0210_{-0.0274}^{+0.0278}$ |
| $\chi^{2} / \mathrm{ndf}$ | 1.5 | 1.5 |
| $N_{\text {bins }}$ | 275 |  |

is in a good agreement with that, extracted from the analysis of our $K_{e 3}$ data [1]: $\lambda_{+}^{e}=$ $0.0293 \pm 0.0015$, i.e our data do not contradict the $\mu-e$ universality.
In the second column the results of the joined fit of our $K_{e 3}$ and $K_{\mu 3}$ data are presented(this is practically equivalent to fixing the $\lambda_{+}$to it's $K_{e 3}$ value). This fit, of course, assumes the $\mu-e$ universality. The $\lambda_{0}$ value $\lambda_{0}=0.0209 \pm 0.0042$ is in a good agreement with the calculations in the framework of the chiral perturbation theory $(\chi P T)[6]: \lambda_{0}^{t h}=0.017 \pm 0.004$.
All the errors presented are from the "MINOS" procedure of the "MINUIT" program [9] and are larger than the Gaussian ones. At present, we estimate an additional systematics error in $\lambda_{+}, \lambda_{0}$ to be $\pm 0.002$. The estimate is done by varying cuts, cell size during the fit of the Dalitz plots etc.

In the second and in the third lines the scalar and the tensor terms are added into the fit. As it is seen from the second line of formula (2), the $f_{S}$ term is $100 \%$ anti-correlated with the V-A contribution $\left(m_{\mu} / 2 m_{K}\right) f_{-}$, i.e an independent estimate of $f_{-}$is necessary to extract $f_{S}$. By definition, $f_{-}=f_{+}(0)\left(\lambda_{0}-\lambda_{+}\right) \frac{m_{K}^{2}-m_{\pi}^{2}}{m_{\pi}^{2}}$. $\lambda_{+}$is, in fact, defined by the $K_{e 3}$ data, and $\lambda_{0}$ is calculated by $\chi P T$ : $\lambda_{0}^{t h}=0.017 \pm 0.004$. In our $f_{S}$ fit we fix $\lambda_{0}$ to this, theoretical, value. The error $( \pm 0.004)$ in the theoretical prediction induces an additional error in $f_{S}$ equal to $\pm 0.005$.

A possible example of theories, which give nonzero $f_{S}$ are the 2 HDM 10 and the Weinberg $3 H D M$ model [11]. In these theories, $f_{S}$ comes from the diagram with the charged Higgs boson
exchange $\mathrm{H}^{-}$. The calculation of the contributions gives [12]:

$$
\begin{array}{r}
f_{S}^{2 h d m} / f_{+}(0)=\frac{m_{\mu}}{2 m_{K}} \cdot \frac{m_{K}^{2}}{m_{H}^{2}} \cdot \operatorname{tg}^{2}(\beta) \\
f_{S}^{3 h d m} / f_{+}(0)=\frac{m_{\mu}}{2 m_{K}} \cdot \frac{m_{K}^{2}}{m_{H_{1}}^{2}} \cdot \operatorname{Re}\left(\alpha_{1}^{*} \gamma_{1}\right) \tag{5}
\end{array}
$$

Here $m_{H}$ is the charged Higgs-boson mass (mass of the lightest $H^{ \pm}$in case of 3 HDM ); $\operatorname{tg}(\beta)=$ $v_{2} / v_{1}$ - the ratio of the vacuum expectation values for 2 Higgs doublets; $\alpha$ and $\gamma$ are complex couplings of the 3HDM Higgs boson to d-quarks and leptons.
From our limit for $f_{S}$ :

$$
\begin{gathered}
\frac{\operatorname{tg}(\beta)}{m_{H}}=0.39 \pm 0.2(\text { stat }) \pm 0.2(\text { theory }) \mathrm{GeV}^{-1} \\
\operatorname{Re}\left(\alpha_{1}^{*} \gamma_{1}\right) \frac{m_{K}^{2}}{m_{H_{1}}^{2}}=0.036 \pm 0.047(\text { stat }) \pm 0.047(\text { theory })
\end{gathered}
$$

Our 2HDM limit is comparable with that from LEP searches for the decay $b \rightarrow \tau \nu_{\tau}$ [13]: $90 \%$ C.L. limit is $\frac{\operatorname{tg}(\beta)}{m_{H}}<0.4 \div 1 \mathrm{GeV}^{-1}$ (depending on collaboration).

The results of the fit with the tensor term are presented in the third line. The tensor term is also correlated with $\lambda_{0}, d f_{T} / d \lambda_{0}=-3.5$. That's why we decided to apply the same approach for the tensor term as for the scalar one, i.e $\lambda_{0}$ is fixed to it's theoretical value and the induced error in $f_{T}$, due to the theoretical error in $\lambda_{0}$ is calculated. The error equals $\pm 0.02$ for the single $K_{\mu 3}$ fit and $\pm 0.014$ for the combined one.
The tensor coupling $f_{T}$ appears naturally in the leptoquark models, as a result of the Fierz transformation [12]. Unfortunately, we have not found complete theoretical consideration for this contribution.

## 6 Summary and conclusions

The $K_{\mu 3}^{-}$decay has been studied using in-flight decays of $25 \mathrm{GeV} K^{-}$, detected by "ISTRA+" magnetic spectrometer. Due to the high statistics, adequate resolution of the detector and good sensitivity over all the Dalitz plot space, the measurement errors are significantly reduced as compared with the previous measurements. The $\lambda_{+}^{\mu}$ parameter of the vector formfactor $f_{+}(t)$ is measured to be:

$$
\lambda_{+}^{\mu}=0.0321 \pm 0.004(\text { stat }) \pm 0.002(\text { syst })
$$

and is in agreement with that obtained from our $K_{e 3}^{-}$data:

$$
\lambda_{+}^{e}=0.0293 \pm 0.0015(\text { stat }) \pm 0.002(\text { syst })
$$

The combined fit of both sets of data assuming the $\mu-e$ universality gives:

$$
\lambda_{+}=0.0296 \pm 0.0014(\text { stat }) \pm 0.002(\text { syst })
$$

The $\lambda_{0}$ parameter of the scalar formfactor $f_{0}(t)$ is measured to be:

$$
\lambda_{0}=0.0209 \pm 0.004(\text { stat }) \pm 0.002(\text { syst }) .
$$

It is, at present, the best measurement of this parameter. It is in a good agreement with $\chi P T$ prediction.

The limits on the possible scalar and tensor couplings are derived:

$$
\begin{gathered}
f_{S} / f_{+}(0)=0.0039 \pm 0.005 \text { (stat) } \pm 0.005 \text { (theory) } ; \\
f_{T} / f_{+}(0)=-0.021 \pm 0.028 \text { (stat) } \pm 0.014 \text { (theory) }
\end{gathered}
$$

The second(theoretical) error comes from the uncertainty in the $\chi P T$ prediction for $\lambda_{0}$. Again, this is the current best estimates for these parameters.

We would like to thank V.V. Braguta, A.E. Chalov, A.A. Likhoded, A.K. Likhoded, R.N. Rogaliov, S.R. Slabospitsky for the discussions.

The INR part of the collaboration is supported by the RFFI fund contract N00-02-16074.

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