A remark on the Primakoff effect

Göran Fäldt*

Department of physics and astronomy,

Uppsala University, Box 516, S-751 20 Uppsala,Sweden

Abstract

The coherent-nuclear reaction $a + \mathrm{A} \to a^\star + \mathrm{A}$ is in the small-angle region dominated by the one-photon-exchange mechanism, often referred to as the Primakoff effect. In this region information about the electromagnetic decay $a^\star \to a + \gamma$ can be obtained. Well-known examples are the two-photon decays of the pi- and eta-mesons. Also decays of charged hadrons can be studied. For charged hadrons the one-photon-exchange amplitude comes with a Coulomb-phase factor and a Coulomb-form factor, which depend on the ratio between transverse- and logitudinal-momentum transfers, the latter being fixed. At the peak of the cross-section distribution, where the two momentum transfers are equal, the form factor could cut down the cross-section value by as much as 40%. Consequently, a determination of a radiative-decay rate that relies on the peak value becomes sensitive to a proper treatment of the Coulomb-form factor.

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*Electronic address: goran.faldt@fysast.uu.se

Many radiative-hadronic-decay rates have been determined through the Primakoff effect. The theory for the Primakoff effect, i.e. coherent-nuclear production in the one-photon-exchange approximation, is laid out in ref.[1]. In the case of charged hadrons the formalism was extended in ref.[2] to include elastic-Coulomb scattering. There, also the appropriate treatment of the coherent-nuclear background was described. The point we would like to make here is that starting from the formalism of [2], the amplitude for a point-like nucleus can be calculated analytically. We shall then see that for charged hadrons the point-like-Coulomb amplitude is adorned with a form factor, extremely important in the vicinity the Primakoff peak.

The amplitude of the proton reaction, $a(k)+p(p) \to a^\star(k')+p(p')$, can in the one-photon-exchange approximation be written as

$$F_p(\mathbf{q}_\perp, q_\|) = 2\alpha \frac{\mathbf{c}\cdot\mathbf{q}_\perp}{\mathbf{q}_\perp^2 + q_\|^2}, \tag{1}$$

where $\mathbf{c}$ is a vector in the impact-parameter plane, defined as the plane orthogonal to the incoming momentum $\mathbf{k}$. The components of the momentum transfer $\mathbf{q}$ are defined as $(\mathbf{q}_\perp, q_\|) = \mathbf{k'} - \mathbf{k}$. The longitudinal component $q_\|$ is fixed, and given by the expression

$$q_\| = -(m_{a^\star}^2 - m_a^2)/2k. \tag{2}$$

The Coulomb-production potential is proportional to $\mathbf{c}\cdot\mathbf{e}(\mathbf{r})$, where $\mathbf{e}(\mathbf{r})$ represents the electric field of the proton. The proton amplitude of Eq.(1) is in fact the Born approximation of this potential, i.e.

$$F_p(\mathbf{q}) = \frac{-\alpha}{2\pi i}\int \mathrm{d}^3 r e^{-i\mathbf{q}\cdot\mathbf{r}}\ \frac{\mathbf{c}\cdot\mathbf{r}}{r^3}, \tag{3}$$

with $\mathbf{r}/r^3$ the electric field of the proton-point-charge distribution.

For a nuclear target the production takes place in the Coulomb field of the nucleus. If we simplify to a point-like nucleus this implies a multiplication of the proton amplitude by a factor of Z. However, if the projectile is charged we must also take into account the distortion of the trajectory due to the elastic-Coulomb scattering. This is done by introducing a Coulomb-phase factor [3],

$$e^{i\chi_C(b)} = \left(\frac{2a}{b}\right)^{i\eta} \tag{4}$$

where a is the cutoff radius in the Coulomb potential. For a negatively charged projectile

$$\eta = 2Z\alpha/v. \tag{5}$$

The velocity v can be put to unity. Thus, Eq.(3) generalized to nuclear scattering becomes

$$F_Z(\mathbf{q}) = \frac{-Z\alpha}{2\pi i} \int \mathrm{d}^3 r e^{-i\mathbf{q}\cdot\mathbf{r}} \, \frac{\boldsymbol{c}\cdot\boldsymbol{r}}{r^3} \left(\frac{2a}{b}\right)^{i\eta}. \tag{6}$$

Integration over the z-variable yields a modified Bessel function, but also the integration over the impact parameter can be performed analytically [4]. We write the result as

$$F_Z(\mathbf{q}) = \mathbf{c}\cdot\mathbf{q} \; F_C(\mathbf{q}), \tag{7}$$

splitting off the off-shell elastic-Coulomb-scattering amplitude $F_C(\boldsymbol{q})$,

$$F_C(\boldsymbol{q}) = \frac{2Z\alpha(aq)^{i\eta}e^{i\sigma_\eta}}{\boldsymbol{q}^2} \; h_C(\boldsymbol{q}) \tag{8}$$

with η defined in Eq.(5) and

$$\sigma_\eta = 2\arg\Gamma(1 - i\eta/2). \tag{9}$$

The extracted phase factors in Eq.(8) are the same as in elastic-Coulomb scattering, except that now

$$q = \sqrt{\boldsymbol{q}_\perp^2 + q_\parallel^2}. \tag{10}$$

In high-energy-elastic scattering the longitudinal-momentum transfer $q_\parallel$ vanishes. In that case q of Eq.(8) is simply $q_\perp$. The Coulomb form factor $h_C(\boldsymbol{q})$ emerges as [4]

$$h_C(\boldsymbol{q}) = \Gamma(2 - i\eta/2)\Gamma(1 + i\eta/2)F(i\eta/2, 1 - i\eta/2; 2; \frac{q_\perp^2}{q_\perp^2 + q_\parallel^2}). \tag{11}$$

In elastic scattering the longitudinal-momentum transfer vanishes, and $h_C(q_\perp, q_\parallel = 0) = 1$ as expected. In Coulomb production with neutral projectiles there is no Coulomb scattering and consequently no Coulomb-form factor, and for charged projectiles $h_C(\mathbf{q}) = 1$ for $q_\perp \gg q_\parallel$, i.e. for transverse-momentum transfers far beyond the Primakoff peak.

The form factor $h_C(\boldsymbol{q})$ is important only in the peak region and there, it is unimportant for light nuclei but extremely important for heavy nuclei. This is illustrated in Fig. 1a where we have plotted the spin-averaged cross section with and without form factor for the lead nucleus. The solid line traces the cross-section distribution ignoring the form factor, and the dashed line the distribution with the form factor $h_C(\boldsymbol{q})$. The normalization is chosen so that the distribution without form factor peaks at one. The variable ζ along the x-axis measures the ratio $\zeta = q_\perp/q_\parallel$.

Obviously, for heavy nuclei, the form factor is very important in the peak region. If it is possible experimentally to map out the peak that is advantageous. But in many situations

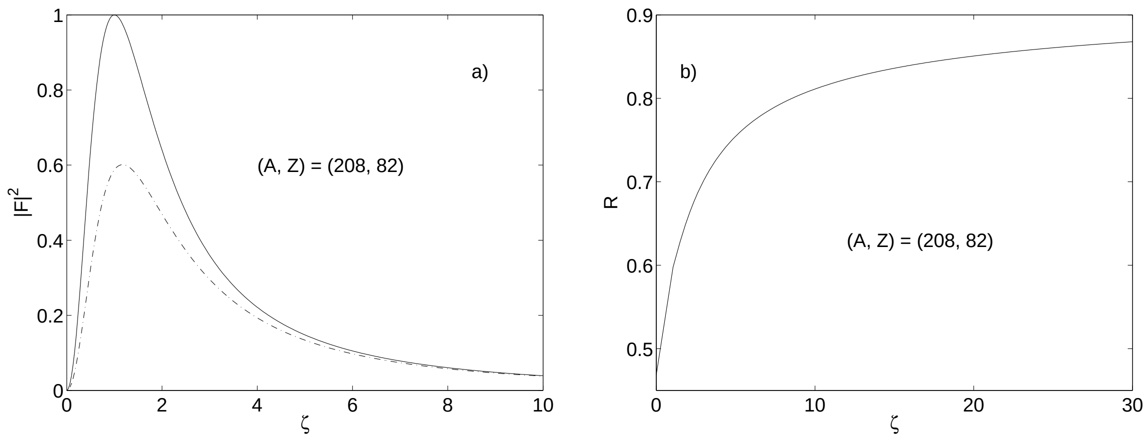

FIG. 1: a) Cross-section distribution in the peak region. Solid line calculated without and dash-dotted line with Coulomb-form factor. The variable $\zeta = q_\perp/q_\|$. b) Ratio R of cross-section distributions integrated up to transverse-momentum transfer ζ. In the numerator of R, the distribution with and in the denominator the distribution without Coulomb-form factor.

the whole peak and even more is contained in the first momentum-transfer bin. In such a case it is more useful to look at the ratio of the cross-section distributions integrated out to some value of momentum transfer. In Fig. 1b the integrated-cross-section ratio

$$R(\zeta) = \left[\int_0^{\zeta^2} \mathrm{d}y \frac{y}{(1+y)^2} |h_C(y)|^2\right] \Big/ \left[\int_0^{\zeta^2} \mathrm{d}y \frac{y}{(1+y)^2}\right] \tag{12}$$

is graphed. As before $\zeta = q_\perp/q_\|$. We conclude that for a lead nucleus the Coulomb-form factor also dominates the integrated cross-section value far beyond the the peak.

In this note we have considered the contribution from the point-like-Coulomb-nuclear-charge distribution. The residual term due to the extension of the charge is easy to calculate numerically once we have extracted analytically the point-charge amplitude. This additional term is small in the region of the peak. Another neglected contribution is the coherent-nuclear-production contribution. It is negligible at high energies but can be important at low energies. These two terms are preferrably handled with the methods of ref.[4].

The Coulomb-form factor is always important in the peak region. Nevertheless, there are experiments at high energies where it seems to have been ignored [5, 6], but it is not clear how that omission affects the radiative-decay rates extracted. On the other hand, there are also experiments both at high [7] and low energies [8] where coherent-nuclear production and Coulomb-scattering effects, including the extension of the nuclear-charge distribution, are fully treated in the formalism of [2]. Those analyses would have been much simplified with the analytic form factor of Eq.(11) as a starting point.

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