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HIGH-ENERGY SCATTERING OF PROTONS BY NUCLEI

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Abstract: The theory of high-energy hadron-nucleus collisions is discussed by means of the multiple-diffraction theory. Effects of the Coulomb field are accounted for in elastic scattering by light and heavy nuclei. Inelastic scattering is treated by means of the shadowed single collision approximation at small momentum transfer and the corresponding multiple collision expansion at large momentum transfers. The theory is compared with the measurements of Bellettini et al. on proton-nucleus scattering at 20 GeV/c by finding density distributions for the nuclei which provide least-squares fits to the data. The nucleon densities found are closely comparable in dimensions to the known charge densities. The predicted sums of the angular distributions of elastic and inelastic scattering reproduce the experimental angular distributions fairly closely.

1. INTRODUCTION

An increasing number of experiments has been undertaken in recent years to study the scattering or production of high-energy particles in nuclei. The electron scattering experiments, which are among the earliest of these, furnish an accurate determination of the nuclear charge distribution. The use of protons or pions as projectiles in high-energy nuclear scattering experiments has, on the other hand, hardly been more than begun. We shall try to show in the present paper that such experiments can furnish a determination of the density distributions of nucleons comparable in accuracy with the known charge distributions.

High-energy data on hadron scattering and production processes in nuclei are conveniently analyzed by means of the multiple diffraction theory of Glauber [1,2]. The application of the multiple diffraction theory to data on unstable particle production, for example, makes it possible to evaluate the unstable particle-nucleon cross section [3]. But such applications of the theory require knowledge of the nucleon density distributions in nuclei, and

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present uncertainties regarding those distributions lead to large uncertainties in the inferred cross sections. It would be of great help to various areas of elementary particle physics, therefore, if these densities could be established more accurately.

We shall illustrate the application of the multiple diffraction theory by analyzing the data of Bellettini et al. [4] on the scattering of 20 GeV/c protons by nuclei. Since the proton-nucleon scattering amplitude is fairly well known at this energy, the ability of the theory to fit the nuclear cross-section data furnishes, in some measure, a test of the accuracy of the theory. Besides checking the theory, an accurate fit furnishes, as we shall show, a determination of the nucleon density function. Several papers [5] containing interpretations of the data of ref. [4] have, in fact, already been published and indicate quite a fair degree of agreement between theory and experiment. We propose in the present paper to base our analysis on a considerably more detailed and extensive application of the underlying theory.

In the experiment of Bellettini et al. [4] the angular distributions of protons scattered by the nuclei ^{6}Li, ^{7}Li, ^{9}Be, C, Al, Cu, Pb and U have been measured in the range from about 2 to 20 mrad. The experimental energy resolution (± 50 MeV) was sufficient for rejecting events in which pion production took place but far from sufficient for isolating elastic scattering. The measured angular distribution represents instead the summed differential cross sections of elastic and inelastic scattering.

In sect. 2 we shall first briefly review the basic expressions of the multiple diffraction approximation. We then give explicitly the formulae used in the calculations for both elastic and inelastic scattering, discussing the approximations which are involved in their derivation. Coulomb scattering, which contributes significantly at the smallest angles, has been taken fully into account. In sect. 3 we present the results of the computations and the fits to the data.

2. THE DIFFRACTION APPROXIMATION

We begin by considering the general form taken by the elastic scattering amplitude for proton-nucleon collision in the diffraction approximation, when spin effects are neglected [1, 2]. If the initial and final momenta of the incident particle are $\hbar \boldsymbol{k}$ and $\hbar \boldsymbol{k}'$, the scattering amplitude may be written as

$$f(\boldsymbol{k}-\boldsymbol{k}') = \frac{ik}{2\pi} \int e^{i(\boldsymbol{k}-\boldsymbol{k}')\cdot \boldsymbol{b}} \Gamma(\boldsymbol{b})\, d^2b \ , \tag{1}$$

where $\boldsymbol{b}$ is the impact vector which lies in the plane perpendicular to $\boldsymbol{k}$. The integration over d^2b is carried out in this plane.

The amplitude f given by eq. (1) is the Fourier transform of the profile function $\Gamma(\boldsymbol{b})$ which may be expressed in terms of the phase-shift function $\chi(\boldsymbol{b})$ as $\Gamma(\boldsymbol{b}) = 1 - e^{i\chi(\boldsymbol{b})}$. The profile function can be derived from f by means of the inverse transform

$$\Gamma(\boldsymbol{b}) = \frac{1}{2\pi ik} \int \mathrm{e}^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(\boldsymbol{q})\, \mathrm{d}^2 q\ .$$

When $\Gamma(\boldsymbol{b})$ depends only upon the magnitude of $\boldsymbol{b}$, the integration over the azimuthal angle gives the Bessel function J_0 and we have

$$f(\boldsymbol{k}-\boldsymbol{k}') = ik \int_0^\infty J_0(|\boldsymbol{k}-\boldsymbol{k}'|\, b)\Gamma(b)\, b\, \mathrm{d}b\ .$$

By comparing this relation with the familiar partial wave expansion of the scattering amplitude, the following correspondence is found to hold: $l+\frac{1}{2} \rightarrow kb$ and $2\delta_l \rightarrow \chi(b)$, where δ_l is the conventional phase shift of the lth partial wave.

The proton-proton scattering amplitude at high energy and small angles can be written, neglecting Coulomb effects for the moment, as

$$f(q) = f(0)\, \mathrm{e}^{-\frac{1}{2}\beta^2 q^2}\ ,$$

where $f(0) = (i+\alpha)k\sigma/4\pi$ and σ is the total cross section. The phase of the scattering amplitude, which has thus far been measured only in the forward direction, is thereby assumed to be constant. Since, according to ref. [6], the proton-proton and proton-neutron amplitudes do not differ significantly at 20 GeV, we shall assume that they are equal.

For the parameters occurring in the amplitude we have used the values, $\sigma = 39.0$ mb, $\beta^2 = 10.0\ (\mathrm{GeV}/c)^{-2}$ and $\alpha = -0.22$ which have been derived from ref. [7] and more recent data [8].

2.1. *Proton-nucleus collisions*

The scattering amplitude for a proton-nucleus collision, if we neglect the effect of the c.m. motion which will be discussed later, may be written as

$$F_{\mathrm{fi}}(\Delta) = \frac{ik}{2\pi} \int \mathrm{e}^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}}\ \psi_{\mathrm{f}}^*(\{\boldsymbol{r}_j\})\Gamma(\boldsymbol{b}, s_1 \ldots s_A)\psi_{\mathrm{i}}(\{\boldsymbol{r}_j\}) \prod_{j=1}^{A} \mathrm{d}^3 r_j\, \mathrm{d}^2 b\ , \qquad (2)$$

where ψ_{i} and ψ_{f} are the initial and final state wave functions of the nucleus and $\boldsymbol{\Delta} = \boldsymbol{k} - \boldsymbol{k}'$. The positions of the A nucleons which make up the target nucleus are defined by the vectors $\boldsymbol{r}_j$, $j = 1, \ldots, A$, and we call s_j the projections of these vectors on the plane perpendicular to $\boldsymbol{k}$ over which the impact vector $\boldsymbol{b}$ is integrated.

In the diffraction approximation the profile function $\Gamma(\boldsymbol{b}, s_1 \ldots s_A)$ is written as

$$\Gamma(\boldsymbol{b}, s_1 \ldots s_A) \equiv 1 - \mathrm{e}^{i\chi(\boldsymbol{b}, s_1 \ldots s_A)} = 1 - \mathrm{e}^{i\sum_{j=1}^{A} \chi_j(\boldsymbol{b}-s_j)}$$

$$= 1 - \prod_{j=1}^{A} [1 - \Gamma_j(\boldsymbol{b}-s_j)]\ .$$

It is assumed in other words that the overall phase-shift function $\chi(\boldsymbol{b}, s_1 \dots s_A)$ is the sum of the phase-shift functions χ_j for collisions with the individual nucleons. Eq. (2) is the basic expression which is used for the calculation of elastic and inelastic scattering of protons by nuclei.

We assume for the present analysis that the ground state of the nucleus can be described by means of the independent particle model, i.e. we neglect all position correlations of the nucleons and write ψ_i as a product wave function. Introducing the single particle densities $\rho_j(r_j)$ we then have

$$|\psi_i(\boldsymbol{r}_1 \dots \boldsymbol{r}_A)|^2 = \prod_{1}^{A}{}_j\, \rho_j(\boldsymbol{r}_j) \ ,$$

where we use the normalization condition

$$\int \rho_j(\boldsymbol{r}_j)\, \mathrm{d}^3 r_j = 1 \ .$$

The form factor of the single particle density $\rho_j(\boldsymbol{r}_j)$ is defined as

$$S_j(\boldsymbol{q}) = \int \mathrm{e}^{i\boldsymbol{q}\cdot\boldsymbol{r}_j}\, \rho_j(\boldsymbol{r}_j)\, \mathrm{d}^3 r_j \ .$$

The overall density distribution of the nucleus, as given by the sum of the single particle densities, is indicated by $\rho(r)$ and $S(q)$ will be the corresponding nuclear form factor. The normalization of the nuclear density distribution is then

$$\int \rho(r)\, \mathrm{d}^3 r = A \ ,$$

so that $S(0) = A$.

In the present calculations we shall use, for the nuclear density, the same models as have been widely employed in the analysis of high-energy electron scattering experiments [9]. For the light elements (Li, Be and C) we use the density which corresponds to the harmonic oscillator potential well. The single particle densities $\rho_s(r)$ and $\rho_p(r)$ for the s- and p-shell are given by

$$\rho_s(r) = \frac{1}{\pi^{\frac{3}{2}} a_o^3}\, \mathrm{e}^{-r^2/a_o^2} \ , \qquad \rho_p(r) = \frac{2}{3\pi^{\frac{3}{2}} a_o^5}\, r^2\, \mathrm{e}^{-r^2/a_o^2} \ ,$$

where a_o is the radial parameter. The corresponding form factors are

$$S_s(q) = \mathrm{e}^{-\frac{1}{4}a_o^2 q^2} \quad \text{and} \quad S_p(q) = (1 - \tfrac{1}{6}a_o^2 q^2)\, \mathrm{e}^{-\frac{1}{4}a_o^2 q^2} \ .$$

The total nuclear density is

$$\rho(r) = \frac{4}{\pi^{\frac{3}{2}} a_o^3} \left(1 + \delta\, \frac{r^2}{a_o^2}\right) \mathrm{e}^{-r^2/a_o^2} \ ,$$

where $\delta = \frac{1}{6}(A-4)$.

The rms radius of the harmonic oscillator well density distribution is given by $\langle r^2\rangle^{\frac{1}{2}} = a_0(\frac{5}{2} - 4/A)^{\frac{1}{2}}$ and its overall form factor is

$$S(q) = A(1 - \frac{\delta}{A} a_0^2 q^2)\, e^{-\frac{1}{4}a_0^2 q^2} .$$

For the nuclei Al, Cu, Pb and U we use the Woods-Saxon density

$$\rho(r) = \rho_0 \left[\exp\left(\frac{r-c}{\tau}\right) + 1\right]^{-1} ,$$

where ρ_0 is the normalization constant, c is the 'half-density' radius and the parameter τ is related to the 'surface thickness' t by $t = 4\tau \ln 3 = 4.40\tau$. Approximate expressions for ρ_0 and for the rms radius of this density have been given by Elton [10].

2.2. *Elastic nuclear scattering*

By using eq. (2) and the factorization assumptions of the preceding section we find that the elastic scattering amplitude of a nucleus in absence of Coulomb forces is given by [1, 2]

$$F_N(\Delta) = \frac{ik}{2\pi} \int e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \{1 - \prod_1^A{}_j [1 - \int \rho_j(r_j)\Gamma(\boldsymbol{b} - \boldsymbol{s}_j)\, d^3r_j]\}\, d^2b$$

$$= \frac{ik}{2\pi} \int e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \{1 - \prod_1^A{}_j [1 - \frac{1}{2\pi ik} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(q) S_j(q) d^2q]\}\, d^2b , \tag{3}$$

where $\Gamma(b)$ and $f(q)$ are the profile function and scattering amplitude, respectively, for proton-nucleon collisions as discussed in sect. 2.

For the harmonic oscillator density and $4 \leqslant A \leqslant 16$, eq. (3) leads to the amplitude

$$F_N(\Delta) = \frac{ik}{2\pi} \int e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \{1 - [1 - \frac{1}{2\pi ik} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(q) S_s(q) d^2q]^4$$
$$\times [1 - \frac{1}{2\pi ik} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(q) S_p(q) d^2q]^{A-4}\}\, d^2b .$$

As a consequence of the gaussian form taken by $f(q)$, the integrations over the q-variables can be performed analytically and the result written in the form

$$F_N(\Delta) = ik \int_0^\infty J_0(\Delta b)\, \{1 - [1 - (1 - i\alpha) G_s(b)]^4 [1 - (1 - i\alpha) G_p(b)]^{A-4}\}\, b db . \tag{4}$$

The explicit expressions for $G_s(b)$ and $G_p(b)$ are given in the appendix.

By introducing a suitable nuclear phase-shift function $\chi_N(b)$, eqs. (3) and (4) can be written in the form

$$F_N(\Delta) = ik \int_0^\infty J_0(\Delta b)\, [1 - e^{i\chi_N(b)}]\, b db . \tag{5}$$

When A is large it is not difficult to show [1, 2] that the effective nuclear phase-shift function is accurately approximated by

$$i\chi_N(b) = -\int \rho(r)\Gamma(\boldsymbol{b}-\boldsymbol{s})\mathrm{d}^3r = -\frac{1}{2\pi ik}\int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(q)S(q)\mathrm{d}^2q \,, \tag{6}$$

as long as nucleon position correlations are negligible. We shall refer to this approximation, which we have used for the nuclei Al, Cu, Pb and U as the large-A approximation. For the light elements on the other hand, our elastic scattering calculations have been based on eq. (4). The total cross section for proton-nucleus scattering has been calculated from the imaginary part of the forward scattering amplitude by means of the optical theorem.

If the nuclear radius is sufficiently large compared to the range of nuclear forces the form factor $S(q)$ will be quite sharply peaked near the forward direction, $q = 0$. The integral given by eq. (6) will then be proportional to $f(0)$, and the corresponding expression for the elastic scattering amplitude (5) will reduce to

$$F_N(\Delta) = ik\int_0^\infty J_0(\Delta b)\left[1-e^{(2\pi i/k)f(0)T(b)}\right] b\mathrm{d}b \,, \tag{7}$$

where the thickness function $T(b)$ is defined by $T(b) = \int \rho(\boldsymbol{b}+\hat{\boldsymbol{k}}z)\,\mathrm{d}z$, $\hat{\boldsymbol{k}}$ being the unit vector along the direction of $\boldsymbol{k}$. It is the expression (7) which is most frequently identified with the nuclear optical model. Since the nuclear radius is not in fact very much larger than the force range, particularly for the lighter nuclei, we have based our calculations on eqs. (4) and (6) instead.

2.3. *Effects of the Coulomb interaction*

We now consider the way in which the Coulomb interaction influences the elastic scattering. The Coulomb phase-shift function for a nuclear charge distribution may be defined in the diffraction approximation by introducing an (arbitrarily large) atomic screening radius R_{scr} beyond which the field is assumed to vanish. (The value of this screening radius, as we shall see, does not influence the cross section.) The overall phase shift produced by the screened Coulomb field of a nucleus of atomic number Z, is then the sum of two terms which we call $\chi_c(b)$ and χ_{scr}. These phases are given by [11]

$$\chi_c(b) = \frac{2Ze^2}{\hbar v}\left[\log b\int_0^b T_c(b')2\pi b'\,\mathrm{d}b' + \int_b^\infty T_c(b')\log b'\,2\pi b'\,\mathrm{d}b'\right]$$

and

$$\chi_{\mathrm{scr}} = -\frac{2Ze^2}{\hbar v}\log(2R_{\mathrm{scr}}) \,,$$

where v is the incident particle velocity and $T_c(b)$ is the thickness function corresponding to the nuclear charge distribution $\rho_c(r)$, normalized to $\int \rho_c(r)\mathrm{d}^3r = 1$.

In the limit of a point-like charge distribution the phase $\chi_c(b)$ tends to the value

$$\chi_{pt}(b) = \frac{2Ze^2}{\hbar v} \log b \ ,$$

which is just the expression for the Coulomb phase of a point charge given by the diffraction approximation [1]. This Coulomb phase function corresponds to the conventional Coulomb phase shift δ_l which, in the limit of large l, can be written as

$$\delta_l = \frac{Ze^2}{\hbar v} \log (l + \tfrac{1}{2}) \ .$$

The elastic scattering amplitude due to both the nuclear interaction and the screened Coulomb field is obtained by adding together the corresponding phase functions [12],

$$F_{el}(\Delta) = ik \int_0^\infty J_0(\Delta b) \{1 - e^{i[\chi_N(b)+\chi_c(b)+\chi_{scr}]}\} b db \ .$$

It is convenient to separate from this amplitude the expression for scattering by a point charge. We can do that by adding and subtracting the quantity $1 - \exp[i\chi_{pt}(b) + i\chi_{scr}]$ in the integrand. After a rearrangement of the terms we then find [11] for $\Delta \gg R_{scr}^{-1}$

$$e^{-i\chi_{scr}} F_{el}(\Delta) = F_c(\Delta) + ik \int_0^\infty J_0(\Delta b) \{e^{i\chi_{pt}(b)} - e^{i[\chi_N(b)+\chi_c(b)]}\} b db \ , \qquad (8)$$

where $F_c(\Delta)$ is the Coulomb scattering amplitude for a point nucleus and is given by

$$F_c(\Delta) = - \frac{2Ze^2 k}{\hbar v \Delta^2} e^{i\varphi_c} \ ,$$

where

$$\varphi_c = - \frac{2Ze^2}{\hbar v} [C + \log (\Delta/2k)]$$

and C is the Euler constant ($C = 0.577$).

The phase factor, which multiplies $F_{el}(\Delta)$ in eq. (8) is the only result of the calculation which depends on the screening radius R_{scr}. It remains unobserved, of course, in all measurements of nuclear scattering intensities.

For the heavier nuclei, $A \gg 1$, the phase-shift function $\chi_N(b)$ which occurs in eq. (8) can be taken from the approximation given by eq. (6). When A is not large, on the other hand, the expression used for $\chi_N(b)$ must correspond to the one implicit in eq. (3). For light nuclei, in other words, the combined nuclear and Coulomb scattering is given for $\Delta \gg R_{scr}^{-1}$ by

$$e^{-i\chi_{\rm scr}} F_{\rm el}(\Delta) = F_{\rm c}(\Delta) + ik \int_0^\infty J_0(\Delta b)\{e^{i\chi_{\rm pt}(b)}$$

$$- e^{i\chi_{\rm c}(b)} \prod_{1}^{A}{}_j \left[1 - \frac{1}{2\pi ik} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} f(q) S_j(q) \mathrm{d}^2 q\right]\} \mathrm{d}^2 b \ . \qquad (9)$$

For the harmonic-oscillator-density model of the light nuclei the values of the q-integrals in this expression are given in eq. (4) and the appendix.

2.4. *Correction for the center-of-mass correlation*

It has been shown [2] that, when momentum and energy conservation is taken fully into account, the amplitude $F_{\rm fi}(\Delta)$ takes the form

$$F_{\rm fi}(\Delta) = \frac{ik}{2\pi} \int e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b}} \psi_{\rm f}^*(\{\boldsymbol{r}_j\}) \Gamma(\boldsymbol{b}, s_1 \dots s_A) \psi_{\rm i}(\{\boldsymbol{r}_j\}) \delta\Big(\frac{1}{A} \sum_{1}^{A}{}_j \boldsymbol{r}_j\Big) \prod_{1}^{A}{}_j \mathrm{d}^3 r_j \mathrm{d}^2 b \ . \qquad (10)$$

This expression differs from eq. (2) only by the presence of the delta function in the integrand which expresses a constraint on the nuclear c.m. We now consider the effect of that constraint on the elastic scattering. If the densities are of gaussian form, the correct nuclear scattering amplitude $\mathcal{F}_{\rm el}(\Delta)$, i.e. the one which follows from eq. (10) can be written as

$$\mathcal{F}_{\rm el}(\Delta) = R(\Delta) F_{\rm el}(\Delta) \ . \qquad (11)$$

In this expression $F_{\rm el}(\Delta)$ is the approximate amplitude we have discussed in the previous sections, and the correction factor is given by [13]

$$R(\Delta) = e^{\langle r^2 \rangle / 6A \ \Delta^2} \ . \qquad (12)$$

Similarly, for the harmonic oscillator density [13, 14], the correction factor is $R(\Delta) = \exp(a_0^2 \Delta^2 / 4A)$.

No simple analytic correction factor, can be found, however, for much more general forms of the density function. Fortunately we are able to describe the light nuclei, for which the correction is significant by means of the harmonic oscillator density. For the heavier nuclei the corrections given by eq. (12) are presumably only approximate but it remains satisfactory to estimate them in this way since they are numerically quite small.

2.5. *Inelastic scattering*

We are interested here not in the intensity of a specific inelastic process but rather in the total intensity of inelastic scattering. What is measured in the experiment of Bellettini et al. [4] is essentially the quantity $\Sigma_{\rm f} |F_{\rm fi}(\Delta)|^2$, where the sum extends over all final nuclear states, in which no particle production takes place. Included in the sum, of course, is the state f = i which represents elastic scattering. The use, in evaluating this sum, of the completeness property for the set of final states of the nuclear system leads to an expression for the sum which contains only the ground state wave function [1]. If we then subtract the elastic differential cross section, we obtain the total inelastic intensity which can be written as

$$\sum_{f\neq i} |F_{fi}(\Delta)|^2 = \left(\frac{k}{2\pi}\right)^2 \int e^{i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')} \left\{ \prod_{1}^{A}{}_j \left[1 - M_j(\boldsymbol{b}) - M_j^*(\boldsymbol{b}') + \Omega_j(\boldsymbol{b},\boldsymbol{b}')\right] - \prod_{1}^{A}{}_j \left[(1-M_j(\boldsymbol{b}))(1-M_j^*(\boldsymbol{b}'))\right] \right\} \mathrm{d}^2 b\, \mathrm{d}^2 b' \,, \tag{13}$$

where we have used the notation

$$M_j(\boldsymbol{b}) = \int \rho_j(r_j)\Gamma(\boldsymbol{b}-\boldsymbol{s}_j)\mathrm{d}^3 r_j$$

for an integral of a type already considered in sect. 2.2, and we have defined

$$\Omega_j(\boldsymbol{b},\boldsymbol{b}') = \int \rho_j(r_j)\Gamma(\boldsymbol{b}-\boldsymbol{s}_j)\Gamma^*(\boldsymbol{b}'-\boldsymbol{s}_j)\mathrm{d}^3 r_j \,.$$

The cross section given by eq. (13) is best evaluated by making certain further approximations. It is worth noting first that the functions $M_j(\boldsymbol{b})$ and $\Omega_j(\boldsymbol{b},\boldsymbol{b}')$ are in general a good deal smaller than unity in magnitude. If we call the interaction range a and the nuclear radius R, then the functions M_j and Ω_j are typically not larger in magnitude than a^2/R^2. For the larger nuclei these functions assume numerical significance only when their effects are summed over many nucleons. Another property of the functions Ω_j worth noting is their short-range character. Unlike the functions $M_j(\boldsymbol{b})$ which vanish only for $b \gtrsim R+a$, the functions $\Omega_j(\boldsymbol{b},\boldsymbol{b}')$ vanish unless the impact vectors $\boldsymbol{b}$ and $\boldsymbol{b}'$ are within about two interaction ranges of one another, $|\boldsymbol{b}-\boldsymbol{b}'| \lesssim 2a$.

For sufficiently large values of the momentum transfer Δ it is clear that the integrand of eq. (13) will oscillate rapidly unless the impact vectors $\boldsymbol{b}$ and $\boldsymbol{b}'$ have nearly the same component along $\boldsymbol{\Delta}$. In particular for $\Delta \gg R^{-1}$ it becomes possible to approximate the integral in eq. (13) by dropping from the integrand some terms of order a^2/R^2, provided we are careful to retain all of the terms which can lead to effects of order Aa^2/R^2. With such an approximation eq. (13) can be simplified to the form,

$$\sum_{f\neq i} |F_{fi}(\Delta)|^2 = \left(\frac{k}{2\pi}\right)^2 \int e^{i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')} \prod_{1}^{A}{}_j \left\{[1-M_j(\boldsymbol{b})][1-M_j^*(\boldsymbol{b}')]\right\} \times \left\{ \prod_{1}^{A}{}_j \left[1+\Omega_j(\boldsymbol{b},\boldsymbol{b}')\right] - 1 \right\} \mathrm{d}^2 b\, \mathrm{d}^2 b' \,.$$

Since the functions M_j and Ω_j are small in magnitude it is correct for $A \gg 1$ and fairly accurate even for A as small as 6 to replace the A-fold products by exponential functions. We have then [2] for $\Delta \gg R^{-1}$

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = \left(\frac{k}{2\pi}\right)^2 \int e^{i\boldsymbol{\Delta} \cdot (\boldsymbol{b}-\boldsymbol{b}')} e^{i[\chi_N(\boldsymbol{b})-\chi_N^*(\boldsymbol{b}')]} [e^{\Omega(\boldsymbol{b},\boldsymbol{b}')} - 1] \, d^2b \, d^2b' , \tag{14}$$

in which we have written

$$\Omega(\boldsymbol{b},\boldsymbol{b}') = \sum_{1}^{A}{}_j \, \Omega_j(\boldsymbol{b},\boldsymbol{b}')$$

and, according to eq. (6),

$$i\chi_N(\boldsymbol{b}) = -\sum_{1}^{A}{}_j \, M_j(\boldsymbol{b}) \; .$$

Some further insight into the distribution of inelastic scattering for large Δ can be obtained by assuming at this point that the nuclear radius is a good deal larger than the force range. In that case, by exploiting the short-ranged character of the functions $\Gamma(\boldsymbol{b})$ and $\Omega(\boldsymbol{b}, \boldsymbol{b}')$ in comparison with the long-ranged character of the functions $\rho(r)$ and $\chi_N(\boldsymbol{b})$, we can reduce the differential cross section for inelastic scattering to the form [2]

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = \left(\frac{k}{2\pi}\right)^2 \int e^{i\boldsymbol{\Delta} \cdot \boldsymbol{b} - \sigma T(B)} \times \left\{ \exp\left[\frac{T(B)}{k^2} \int e^{-i\boldsymbol{q}\cdot\boldsymbol{b}} \, |f(q)|^2 \, d^2q\right] - 1 \right\} d^2b \, d^2B , \tag{15}$$

In which $T(B)$ is again the thickness function defined in sect. 2.2. This expression for the inelastic cross section is most easily evaluated by expanding the exponential function within the curly brackets. The resulting series, which takes the form of a species of multiple scattering expansion [1, 2], is given by

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = N_1 |f(\Delta)|^2 + \frac{N_2}{\sigma} \int |f(\boldsymbol{q})|^2 \, |f(\boldsymbol{\Delta} - \boldsymbol{q})|^2 \, \frac{d^2q}{k^2} + \ldots , \tag{16}$$

where the dimensionless coefficients N_n are defined as

$$N_n = \frac{1}{n!\sigma} \int e^{-\sigma T(b)} [\sigma T(b)]^n \, d^2b \; . \tag{17}$$

Since each term of the expansion takes the attenuation of the incident beam explicitly into account, the processes described have been referred to as shadowed multiple scattering [11].

Since the attenuation of nucleons travelling centrally through even medium-weight nuclei is quite appreciable, the principal contributions to the integrals N_n come from the surface regions of these nuclei. Measurements of the inelastic scattering for $\Delta \gg R^{-1}$ therefore tend to determine nuclear surface parameters, and of these principally the surface thickness [2].

When the nucleon-nucleon scattering amplitude is given by the Gaussian parametrization introduced in sect. 2, the multiple scattering integrals are easily evaluated and the series (16) is found to take the form

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = |f(0)|^2 \sum_{n=1}^{A} \frac{1}{n} N_n \epsilon^{n-1} e^{-\beta^2 \Delta^2 / n} , \tag{18}$$

in which the expansion parameter ϵ is given by

$$\epsilon = \frac{1+\alpha^2}{16\pi} \frac{\sigma}{\beta^2} .$$

It is worth noting that the angular distributions contributed by the successively higher orders of multiple scattering decrease more and more slowly with increasing momentum transfer. Since for 20 GeV/c incident protons the parameter ϵ has the value 0.21, the series (18) converges fairly rapidly for small values of Δ. For large Δ, however, it is evident that the series will converge somewhat more slowly. Far enough from the forward direction, in fact, the angular distribution will be dominated by double, then triple, then quadruple scattering, etc.

Since the radii of the lighter nuclei are not in fact many times larger than the interaction range it is more accurate to calculate their cross sections by means of eq. (14) than by means of eqs. (16) or (18). The use of eq. (14), however, is practical only if the integrations which are required in order to obtain the functions $\chi_N(\boldsymbol{b})$ and $\Omega(\boldsymbol{b}, \boldsymbol{b}')$ can be carried out analytically. For the case of the harmonic oscillator-well density functions the integrals can indeed be evaluated in closed form. The resulting functions $\chi_N(\boldsymbol{b})$ and $\Omega(\boldsymbol{b}, \boldsymbol{b}')$ are listed in the appendix.

Direct numerical computation shows that for ^{6}Li the cross section given by eq. (18) is about 20% lower than that given by eq. (14). For Al, on the other hand, the difference of the cross sections is only about 3%. We have therefore used eq. (14) to describe the large-angle inelastic scattering by the nuclei lighter than Al, and eq. (18) to describe the scattering by Al, Cu, Pb and U.

We turn next to the description of the inelastic scattering in the region of small momentum transfers, $\Delta \lessapprox R^{-1}$. This is, of course, the region in which the contribution of the elastic scattering to the intensity observed is vastly greater than the contribution of the inelastic scattering. It is not necessary, therefore, to attempt to describe the inelastic scattering as accurately at small angles as we have at large angles where it contributes nearly all of the observed intensity.

We have already noted in connection with eq. (18) that as Δ becomes small, shadowed single collision processes tend to make up nearly all of the inelastic scattering. We can evidently approximate the inelastic scattering at small angles, therefore, by returning to eq. (13) and extracting the shadowed single collision term from it. Since we are no longer assuming $\Delta \gg R^{-1}$ it is not possible to neglect all of the terms of order

a^2/R^2 in the integrand which were neglected earlier. On the other hand many terms may be dropped since they represent second- or higher-order multiple scattering. If we assume that all of the partial density functions $\rho_j(r_j)$ are similar in shape and that $A \gg 1$, then we find the single collision cross section

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = \left(\frac{k}{2\pi}\right)^2 \int e^{i\boldsymbol{\Delta}\cdot(\boldsymbol{b}-\boldsymbol{b}')} e^{i[\chi_N(\boldsymbol{b})-\chi_N^*(\boldsymbol{b}')]}$$

$$\times \{\Omega(\boldsymbol{b},\boldsymbol{b}') - \frac{1}{A}\chi_N(\boldsymbol{b})\chi_N^*(\boldsymbol{b}')\} d^2b\, d^2b' . \quad (19)$$

When the nuclear radius is quite large compared to the interaction range this expression can be reduced to the form [11]

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = |f(\Delta)|^2 \{N_1 - \frac{1}{A}\left| \int e^{i\boldsymbol{\Delta}\cdot\boldsymbol{b} + i\chi_N(\boldsymbol{b})} T(\boldsymbol{b}) d^2b \right|^2\} . \quad (20)$$

The inelastic scattering, in other words, is a good deal smaller near the forward direction than that due to N_1 free nucleons. As Δ increases beyond the diffraction cone $\Delta \sim R^{-1}$, however, the integral within the curly brackets goes to zero and the inelastic intensity then agrees, as it should, with the first term of the series (16).

If the nucleons were weak scatterers, of course, double and triple scattering would never occur. The expression (20), which should represent the inelastic scattering quite accurately in that case, reduces to the form

$$\sum_{f \neq i} |F_{fi}(\Delta)|^2 = |f(\Delta)|^2 A \{1 - \frac{1}{A^2} S^2(\Delta)\} , \qquad (\sigma \to 0) ,$$

which is the familiar result of the impulse approximation [15].

In our numerical calculations we have evaluated the inelastic scattering by correcting the large-momentum transfer formulas so that their single collision term is made accurate for all Δ. We have, in effect, treated the second term in the curly brackets of eq. (20) as a correction to eq. (14) for the lighter nuclei and to eq. (18) for the heavier nuclei.

3. RESULTS OF THE CALCULATIONS

Since we have assumed the proton-nucleon scattering amplitude to be known, the only quantities which can be determined by fitting the experimental data are the parameters of the nuclear density distribution. For the light nuclei, Li, Be and C, there is, in the model based on the harmonic oscillator density functions, only one parameter to be found, a_0 or equivalently the rms radius $\langle r^2\rangle^{\frac{1}{2}}$. For Al, Cu, Pb and U, on the other hand, the use of the Woods-Saxon model for the density function means that the two parameters c and t must be determined.

We have evaluated the angular distributions of elastic scattering numerically by using eqs. (8), (9), (11) and (12), and by assuming in the Coulomb calculations that the nuclear charge distribution has the same shape as the nuclear density distribution. The elastic angular distributions were added to the inelastic ones calculated by means of the formulas noted in sect. 2.5 and the sums compared with the total scattered intensities measured by Bellettini et al. [4]. A least-squares fit to the experimental data for each nucleus was found by varying the appropriate density parameters.

The calculated angular distributions for summed scattering which represent the best fits are shown together with the measured distributions for the eight nuclei used in the experiment in figs. 1-8. The calculated distributions of elastic and inelastic scattering, which are also presented separately in these graphs, show how completely the measured intensities are dominated by elastic collisions at small angles and inelastic ones at large angles.

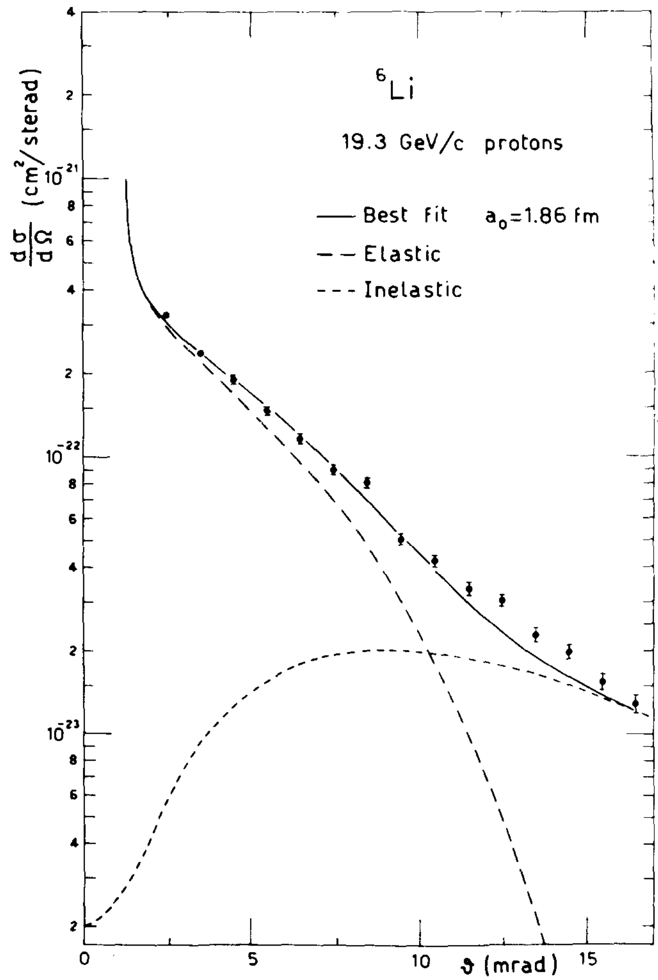

Fig. 1. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by ^{6}Li are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

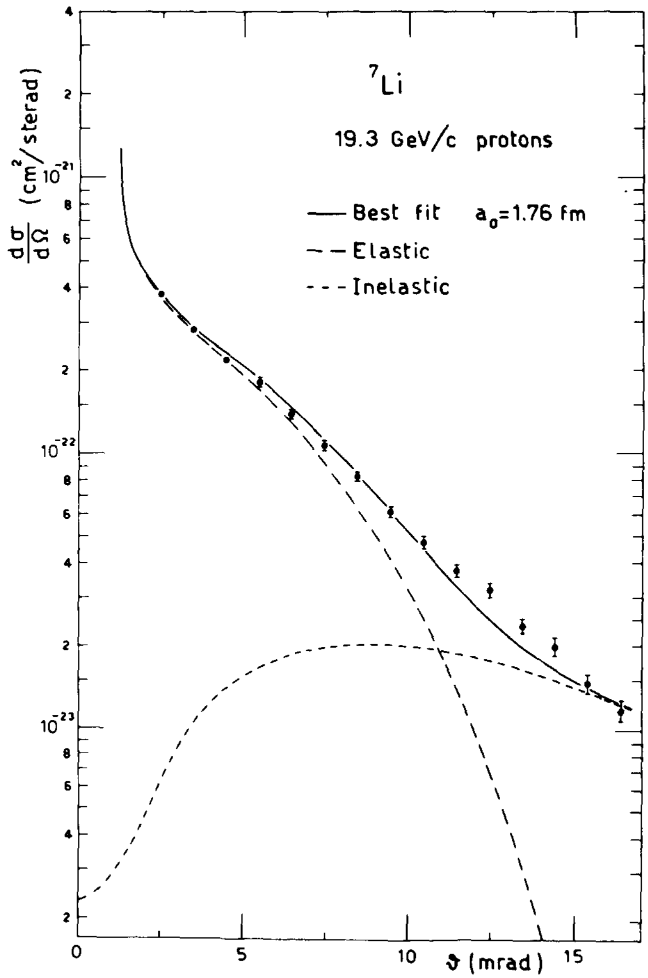

Fig. 2. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by ^{7}Li are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

The calculated distributions do indeed follow the behavior of the measured distributions quite well for all eight nuclei. The minimum χ^2 values which we have obtained are satisfactory and in several cases quite good. The minimum χ^2 value is 116 for ^{6}Li, 96 for ^{7}Li, 72 for ^{9}Be and 30 for C, the number of degrees of freedom being 14. For Al the minimum of χ^2 is 52, which is reduced to 23 if the contribution of the smallest angle data point is excluded. For Cu the minimum of χ^2 is 15. For Pb the minimum of χ^2 is around 400 and for U is larger.

To discuss our results in greater detail let us begin with the light nuclei. For these we have studied the effect of using several of the approximation methods discussed earlier. For ^{6}Li, for example we find that if we

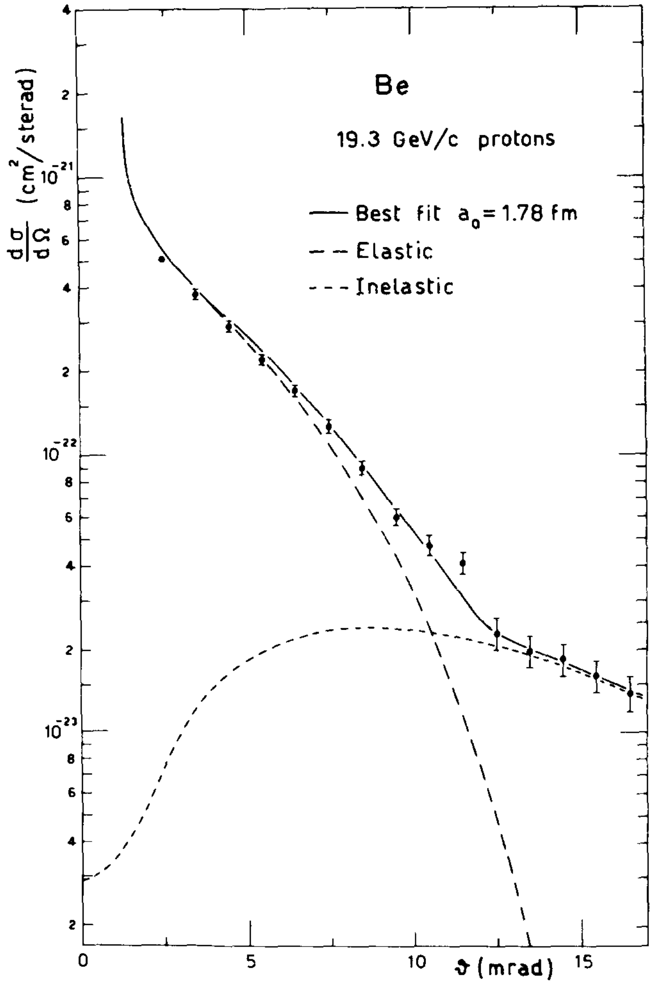

Fig. 3. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by Be are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

use the large-A approximation for the elastic scattering, i.e. eqs. (6) and (8), and the short-range approximation for the inelastic scattering, i.e. eq. (18) the fit is quite poor (the minimum of χ^2 is about 260). When the more accurate expression (14) is used to fit the ^{6}Li data, the minimum of χ^2 decreases to 168 and the scattering in the inelastic region is reasonably well reproduced. A further improvement is obtained by using eq. (9) which is the Coulomb generalization of eq. (4) instead of eqs. (6) and (8) to calculate the elastic scattering. Returning in this way to the formulas accurate for small values of A reduces the minimum value of χ^2 to 116. We have observed similar behavior in the calculations for the case of ^{7}Li.

For Be and C a slight improvement in the description of the inelastic

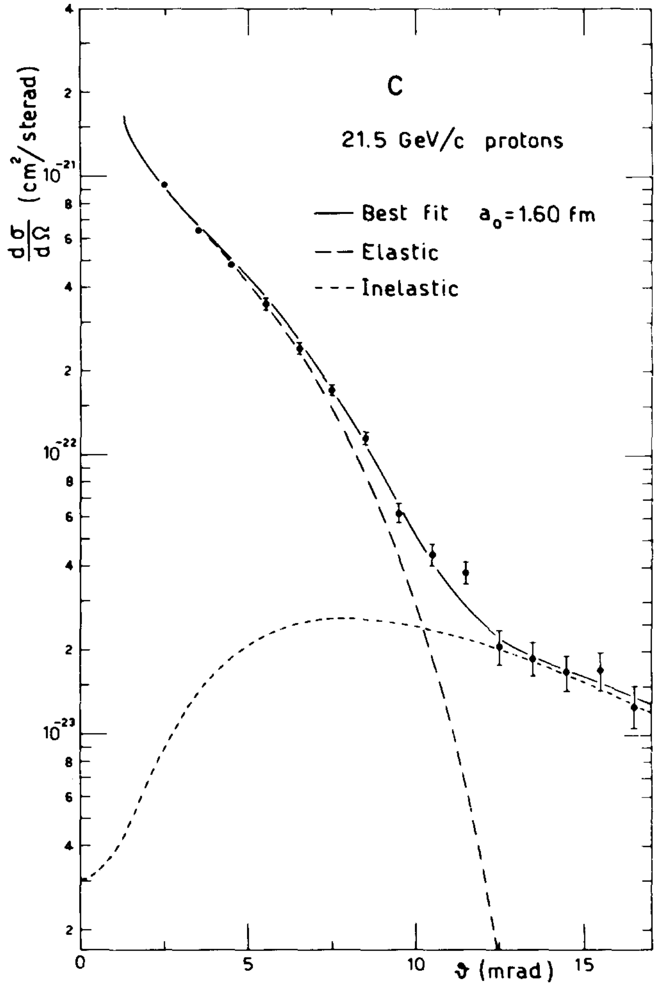

Fig. 4. The experimental data of ref. [4] on the scattering of 21.5 GeV/c protons by C are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

region is obtained by using eq. (14) instead of eq. (18) but there is no substantial difference between the results given by eqs. (4) and (6).

We have listed in table 1 the values of the parameters we have found for the nucleon densities in the light nuclei. The errors assigned to the radial parameters are derived from the criterion $\chi^2 \leq \chi^2_{min} + 1$. For the light nuclei other than carbon we have indicated the errors in parentheses since for these cases the χ^2_{min} values obtained are relatively large. Such errors lack true statistical significance and are only intended to be suggestive. We have also listed for comparison in table 1 the values of the rms radius of the charge distribution found in electron scattering experiments. Our determinations of the rms radius of the nuclear density are seen to corre-

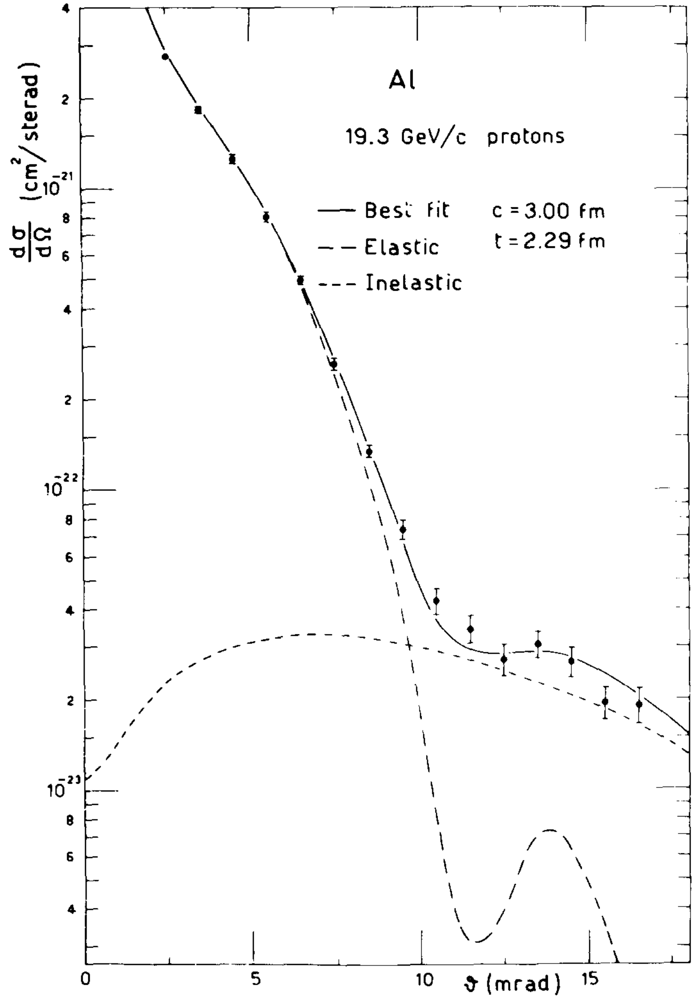

Fig. 5. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by Al are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

spond rather closely to the values obtained for the rms radius of the charge distribution.

In table 2 we have listed the values of the parameters c and t of the Woods-Saxon model which we have found for Al, Cu, Pb and U, together with the results obtained from electromagnetic interactions, i.e. electron scattering and muonic X-ray experiments. In particular for Al and Cu we have obtained excellent fits to the measured angular distributions. For these nuclei furthermore the density parameters we have found correspond rather well to the ones found from electron scattering data. The errors given in table 2 have been derived from the same criterion we have used for table 1.

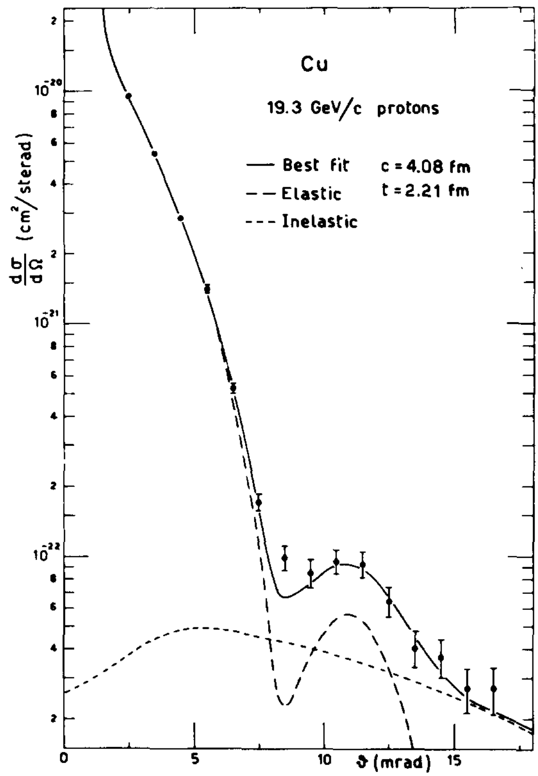

Fig. 6. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by Cu are shown together with the result of the best fit. Elastic and inelastic contributions are shown separately.

For lead our fit is not truly quantitative in the sense that the minimum χ^2 value is quite large. The main contribution to the χ^2 comes from the small-angle data points which have been given a very small statistical error in the experimental data. A computed angular distribution which follows the Pb data more closely away from the forward direction than the one for minimum χ^2 corresponds to a value of the half-density radius c, which is very similar to the one found from electromagnetic interactions. In order to reproduce the experimental data, however, a rather large value of the surface thickness parameter is required, as already observed by Goldhaber and Joachain [5].

For uranium fewer data points are available and our fit to them is

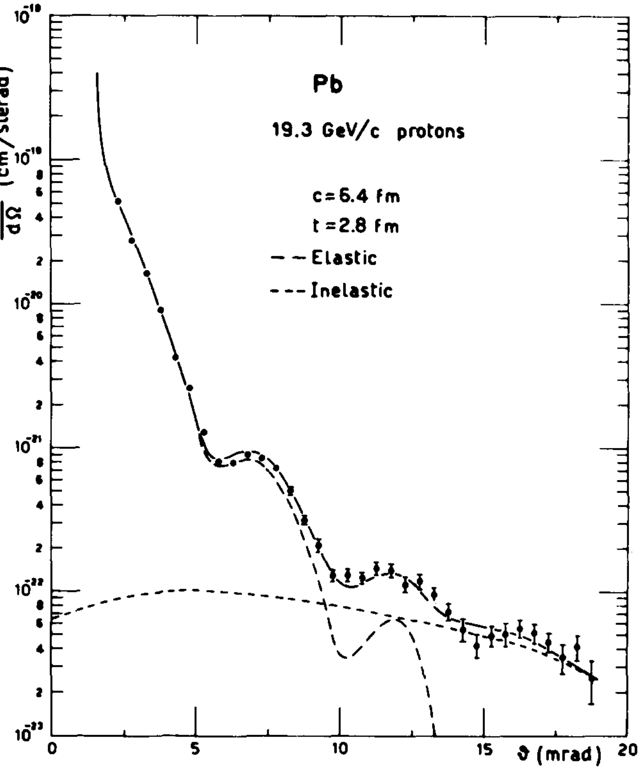

Fig. 7. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by Pb are shown together with the result of the fit represented by the solid line. Elastic and inelastic contributions are shown separately.

somewhat poorer as well. It does not seem possible, in particular, to fit the data points accurately in the region of the first diffraction minimum and of the subsequent maximum. The reason could be related perhaps to an inadequacy of the nuclear model which we have assumed. The uranium nucleus is known to be strongly deformed, a property which is not represented by the spherically symmetric density function we have used. The values for the parameters c and t given for U in table 2 are only intended to be suggestive.

By using the values of the nuclear density parameters which we have found from the fits to the angular distributions, we can calculate the corresponding total cross sections for the various nuclei. In table 3 we have listed the calculated values of the total cross section and the experimental ones as given by Bellettini et al. [4]. The errors assigned to the calculated cross sections are those that correspond to the uncertainties of the density parameters. It is worth noting that the total cross sections given in ref. [4] are not fully determined by experimental means. In fact they are obtained by means of transmission measurements which require an extrapolation to the forward direction that is somewhat dependent on the behaviour assumed

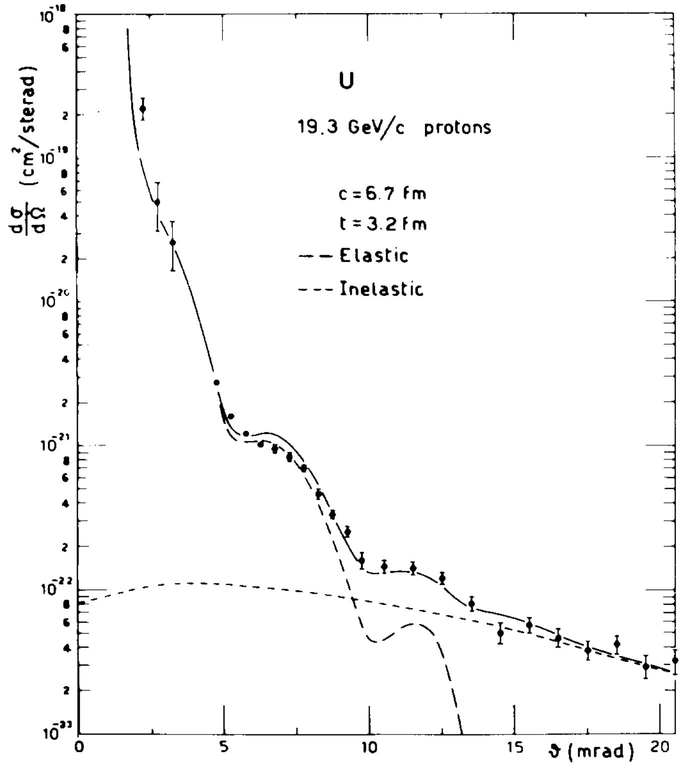

Fig. 8. The experimental data of ref. [4] on the scattering of 19.3 GeV/c protons by U are shown together with the result of the fit represented by the solid line. Elastic and inelastic contributions are shown separately.

for the scattering amplitude at small angles. In ref. [4] the Coulomb contribution to the differential cross section at the small angles was subtracted by assuming the nuclear amplitude to be purely imaginary.

Table 3 shows that the agreement is close for only two or three nuclei. For the Li isotopes and the heavy elements, the total cross sections given in ref. [4] are consistently larger than the predicted ones. We note that if we were to try to determine the nuclear radii from the total cross section values of ref. [4], we would obtain, for lead, for example, a radius about 10% larger than the one given in table 2. Such a large value for the radius, however, would be inconsistent with the experimentally observed angular distribution. It seems then possible that the discrepancies appearing in table 3 could be ascribed to the inadequacy of the extrapolation procedure used in ref. [4] for deriving the total cross sections from the data.

A number of our results have indicated that the diffraction approximation is able to furnish a fairly accurate description of the high-energy scattering of protons by nuclei. If sufficiently accurate versions of the approximation are used, then the angular distributions can be fitted quite well. Such results lend considerable support the procedure of deriving unstable

Table 1

	a_0 from the present analysis (fm)	rms radius from the present analysis (fm)	rms radius from electron scattering (fm)
^{6}Li	1.86 (± 0.03)	2.52 (± 0.04)	2.41 ± 0.07 a) 2.54 ± 0.05 b)
^{7}Li	1.76 (± 0.03)	2.44 (± 0.04)	2.33 ± 0.06 a) 2.39 ± 0.03 c)
^{9}Be	1.78 (± 0.05)	2.55 (± 0.07)	2.60 ± 0.20 d) 2.20 ± 0.20 e) 2.63 ± 0.10 f)
^{12}C	1.60 ± 0.03	2.35 ± 0.04	2.50 ± 0.15 d) 2.42 ± 0.10 e) 2.40 ± 0.02 g)

a) Ref. [16]. Data at low-momentum transfer fitted with the harmonic oscillator model.
b) Ref. [17]. Fit with a phenomenological expression for the form factor.
c) Ref. [17]. Harmonic-oscillator model with electric quadrupole contribution.
d) Ref. [18]. Model-independent value.
e) Ref. [19]. Harmonic-oscillator model.
f) Ref. [19]. Average of several models.
g) Ref. [20].

Table 2

	c (fm)	$c/A^{\frac{1}{3}}$ (fm)	t (fm)	c (fm)	$c/A^{\frac{1}{3}}$ (fm)	t (fm)
	from the present analysis			from electromagnetic interactions		
Al	3.00 ± 0.06	1.00 ± 0.02	2.29 ± 0.07	3.06 3.07 ± 0.09	1.02 1.02 ± 0.03	2.5 a) 2.28 ± 0.11 b)
Cu	4.08 ± 0.04	1.02 ± 0.01	2.21 ± 0.07	4.26	1.06	2.5 a)
Pb	≈ 6.4	≈ 1.08	≈ 2.8	6.5 6.47 ± 0.03 6.67 ± 0.09	1.1 1.091 ± 0.005 1.125 ± 0.015	2.3 c) 2.30 ± 0.03 d) 2.21 ± 0.25 e)
U	(≈ 6.7)	(≈ 1.08)	(≈ 3.2)	7.15 ± 0.03	1.153 ± 0.005	1.46 ± 0.12 f)

a) Values obtained by interpolating with the empirical formula of ref. [21].
b) From electron scattering (ref. [22]).
c) From electron scattering (ref. [19]).
d) From electron scattering (ref. [23]).
e) From muonic X-ray (ref. [24]).
f) From muonic X-ray data analysed with a modified Woods-Saxon model which takes into account the nuclear deformation (ref. [25]).

Table 3

	$\sigma_t(b)$ Calculated	Experimental
^{6}Li	0.203 (± 0.001)	0.232 ± 0.005
^{7}Li	0.229 (± 0.001)	0.250 ± 0.005
^{9}Be	0.278 (± 0.004)	0.278 ± 0.004
C	0.333 ± 0.003	0.335 ± 0.005
Al	0.658 ± 0.010	0.687 ± 0.010
Cu	1.23 ± 0.01	1.36 ± 0.02
Pb	≈ 3.09	3.29 ± 0.10
U	(≈ 3.5)	

particle-nucleon total cross sections, by applying the diffraction approximation to production experiments.

We have also found fairly close agreement between the parameters of the nucleon density distribution, as derived from strong interaction data, and the corresponding parameters of the nuclear charge distribution obtained from electromagnetic interactions. Our results indicate therefore that the neutron distribution in the nucleus cannot be very different in size from the proton distribution.

APPENDIX

For the harmonic oscillator model we have used the following expressions:

$$G_s(b) = \frac{\sigma}{8\pi\gamma^2}\, e^{-\frac{b^2}{4\gamma^2}} ,$$

$$G_p(b) = \frac{\sigma}{8\pi\gamma^2}\left[1 - \frac{a_0^2}{6\gamma^2}\left(1 - \frac{b^2}{4\gamma^2}\right)\right] e^{-\frac{b^2}{4\gamma^2}} ,$$

where

$$\gamma^2 = \tfrac{1}{4}a_0^2 + \tfrac{1}{2}\beta^2 .$$

$$\chi_N(b) = \frac{Af(0)}{2k\gamma^2}\left[1 - \frac{D}{\gamma^2}\left(1 - \frac{b^2}{4\gamma^2}\right)\right] e^{-\frac{b^2}{4\gamma^2}} ,$$

$$\Omega(\boldsymbol{b}, \boldsymbol{b}') = \frac{A\,|f(0)|^2}{4k^2\beta^2\eta^2}\, e^{-\frac{B'^2}{4\beta^2}}\, e^{-\frac{B^2}{4\eta^2}}\left[1 - \frac{D}{\eta^2}\left(1 - \frac{B^2}{4\eta^2}\right)\right] ,$$

where

$$D = \frac{\delta a_0^2}{A}\,, \qquad \eta^2 = \tfrac{1}{4}(a_0^2 + \beta^2)\,, \qquad \boldsymbol{B} = \tfrac{1}{2}(\boldsymbol{b} + \boldsymbol{b}')\,, \qquad \boldsymbol{B}' = \boldsymbol{b} - \boldsymbol{b}'\,.$$

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