Primakoff Production at OKA

Some things to expect and remember

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Outline

What Is Primakoff Effect

Process of electromagnetic production of hadrons in a nucleus Coulomb field at high energy

$$B + (A, Z) \to X + (A, Z)$$

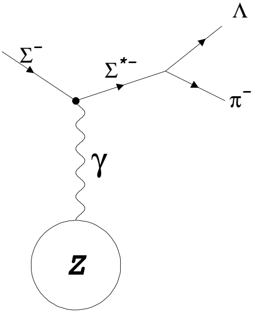
Assumed

- Coherency
- There are processes

$$B + \gamma \to X$$

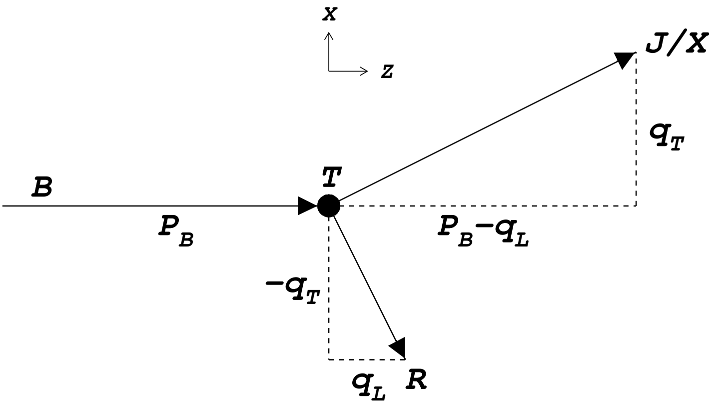
$$J \to B + \gamma$$

Example:
$\Sigma(1385)^-$ production



Kinematics Of Primakoff Production

Laboratory reference frame



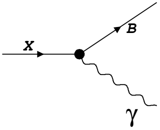
$$q_L = \frac{m_X^2 - m_B^2 + 2q_L^2 + 2q_T^2}{2P_B} \approx \frac{m_X^2 - m_B^2}{2P_B} \qquad q_0 \approx \frac{q_L^2 + q_T^2}{2m_T}$$

General Expressions For Decay And Production

For decay

$$\mathcal{M} = \mathcal{T}_\mu \epsilon^*_\mu$$

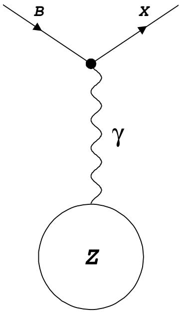
$$\mathrm{d}\Gamma = \frac{(2\pi)^4}{2\mathrm{M}} |\mathcal{M}|^2 \, \mathrm{d}\Phi_\mathrm{n}$$



For Primakoff production

$$\mathcal{M} = \mathrm{eZ}(\mathrm{p}_1 + \mathrm{p}_2)_\mu \, \frac{\mathrm{g}_{\mu\nu}}{\mathrm{q}^2} \, \mathcal{T}^*_\nu$$

$$\mathrm{d}\sigma = \frac{(2\pi)^4}{4\sqrt{(\mathrm{p_B} \cdot \mathrm{p_T})^2 - \mathrm{m_B^2 m_T^2}}} |\mathcal{M}|^2 \, \mathrm{d}\Phi_\mathrm{n}$$



Standard Expressions For Primakoff Production

This is an approximation

Main assumptions

$$m_X \ll P_B$$

$$q_T \ll \left(m_X^2 - m_B^2\right)/(2m_X)$$

If the beam is not a photon

$$\frac{d^2\sigma}{dq_T^2\, dm_X^2} = \frac{\alpha}{\pi} Z^2 \frac{\sigma[B+\gamma \to X]}{m_X^2 - m_B^2} \frac{q_T^2}{(q_L^2 + q_T^2)^2} |F_C(q_T, \ldots)|^2$$

$$\frac{d\sigma}{dq_T^2} = 8\pi\alpha Z^2 \frac{2J_J + 1}{2J_B + 1} \Gamma[J \to B + \gamma] \left(\frac{m_J}{m_J^2 - m_B^2}\right)^3 \frac{q_T^2}{(q_L^2 + q_T^2)^2} |F_C|^2$$

F_C — formfactor — MUST be taken into account

Exact Analytic Calculations For $0^- \to 1^-$

Matrix elements

For decay like $K^*(892)^+ \to K^+ + \gamma$

$$\mathcal{M} = g\, \epsilon_{\mu\nu\lambda\rho}\, \epsilon^*_\mu[\gamma]\, p_\nu[\gamma]\, \epsilon_\lambda[K^*]\, p_\rho[K^*]$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{2J+1} \sum_{\lambda_J} \sum_{\lambda_\gamma} |\mathcal{M}|^2 = g^2 \frac{2}{3} m_J^2 \left(\frac{m_J^2 - m_B^2}{2m_J} \right)^2$$

Thus

$$\mathcal{T}_\mu = g \epsilon_{\mu\nu\lambda\rho} \left(p_\nu[J] - p_\nu[B] \right) \epsilon_\lambda[J]\, p_\rho[J]$$

And for Primakoff production

$$\mathcal{M} = eZ(p_1 + p_2)_\mu \, \frac{g_{\mu\nu}}{q^2}\, \mathcal{T}^*_\nu$$

$$\mathcal{M} = eZ\; 2\, g\, m_T\, P_B\, \frac{q_T}{q^2}\, \epsilon^*_y[\lambda_J]$$

Exact Analytic Calculations For $0^- \to 1^-$

Width and cross section

Decay width

$$\Gamma = \frac{1}{32\pi^2}\frac{|\mathrm{p}|}{\mathrm{m}_\mathrm{J}^2}\int |\mathcal{M}|^2 \, d\Omega$$

$$\Gamma = \frac{1}{12\pi}\mathrm{g}^2|\mathrm{p}|^3 = \frac{1}{12\pi}\mathrm{g}^2\left(\frac{\mathrm{m}_\mathrm{J}^2 - \mathrm{m}_\mathrm{B}^2}{2\mathrm{m}_\mathrm{J}}\right)^3$$

Primakoff production cross section

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s}\frac{1}{|\mathrm{p}_1^{\mathrm{CM}}|^2}|\mathcal{M}|^2$$

Using $\alpha = e^2/(4\pi)$ and $s|\mathrm{p}_1^{\mathrm{CM}}|^2 = \mathrm{m}_\mathrm{T}^2\mathrm{P}_\mathrm{B}^2$

$$\frac{d\sigma}{dt} = \frac{\alpha}{4}\mathrm{Z}^2\mathrm{g}^2\frac{\mathrm{q}_\mathrm{T}^2}{\mathrm{q}^4} = 24\pi\alpha \mathrm{Z}^2\Gamma\left(\frac{\mathrm{m}_\mathrm{J}}{\mathrm{m}_\mathrm{J}^2 - \mathrm{m}_\mathrm{B}^2}\right)^3\frac{\mathrm{q}_\mathrm{T}^2}{\mathrm{q}^4}$$

Exact Analytic Calculations For $0^- \to 1^+$

Matrix elements

For decay like $K_1(1270)^+ \to K^+ + \gamma$

$$\mathcal{M} = g\left(p[\gamma]\cdot\epsilon[K_1]\,p[K_1]\cdot\epsilon^*[\gamma] - p[K_1]\cdot p[\gamma]\,\epsilon[K_1]\cdot\epsilon^*[\gamma]\right)$$

$$\overline{|\mathcal{M}|^2} = \frac{1}{2J+1}\sum_{\lambda_J}\sum_{\lambda_\gamma}|\mathcal{M}|^2 = g^2\frac{2}{3}m_J^2\left(\frac{m_J^2 - m_B^2}{2m_J}\right)^2$$

Thus

$$\mathcal{T}_\mu = g\left(p[\gamma]\cdot\epsilon[K_1]\,p_\mu[K_1] - p[K_1]\cdot p[\gamma]\,\epsilon_\mu[K_1]\right)$$

And for Primakoff production

$$\mathcal{M} = eZ(p_1 + p_2)_\mu\,\frac{g_{\mu\nu}}{q^2}\,\mathcal{T}^*_\nu$$

Exact Analytic Calculations For $0^- \to 1^+$

Matrix elements continued

Exact expression where $\epsilon = \epsilon[\lambda_J]$

$$\begin{aligned}\mathcal{M} &= eZ\,2g\sqrt{m_T^2 + q_L^2 + q_T^2} \times \frac{1}{q^2} \times \\ &\times \left(-\epsilon_0^*(P_B q_L - q_L^2 - q_T^2) - \epsilon_x^* E_X q_T + \epsilon_z^* E_X q_L\right)\end{aligned}$$

Approximations at $q_T \to 0$, $q_L \to 0$

For projections in the Gottfried-Jackson frame

$$\mathcal{M} \approx -eZ\,2\,g\,m_T\,E_X\,\frac{q_T}{q^2} \qquad \text{for} \quad \epsilon(x) = -\frac{\epsilon(+1)+\epsilon(-1)}{\sqrt{2}}$$

$$\mathcal{M} \approx -eZ\,2\,g\,m_T\,m_X\,\frac{q_L}{q^2} \qquad \text{for} \quad \epsilon(z) = \epsilon(0)$$

Formfactor

Some complex number, close to 1 under certain confitions

When the formfactor is 1?

- Small charge
  $\alpha Z_B Z_T \to 0$
- Small nucleus
  $R \to 0$
- Small interaction cross section
  $\sigma_{tot} \to 0$

Total production cross section

$$\frac{d\sigma}{d\Phi} = |f_c\, F_C + f_s\, F_S|^2$$

Formfactor

Usually calculated in Eikonal approximation
by people that measure radiative decay widths

Coherence condition

$$m_X^2 - m_B^2 \lesssim 2P_B/R$$

For $K \to K^*$: $R \lesssim 12.7\,\mathrm{fm}$

$$\begin{aligned} R &\approx 1.12A^{1/3}\,\mathrm{fm} \\ R_C &\approx 2.6\,\mathrm{fm} \\ R_{Cu} &\approx 4.5\,\mathrm{fm} \\ R_{Pb} &\approx 6.6\,\mathrm{fm} \end{aligned}$$

$$F_C(q^2) = \frac{q^2}{q_T}\frac{1}{4\pi i}\int d^3r \; e^{i\vec{q}\,\vec{r}}\,\hat{q_T}\vec{E}(r)\,e^{-\frac{1}{2}\sigma'_B A\int_{-\infty}^{z} dz\,\rho(r)}\,e^{-i\chi_C(r)}\,e^{-\frac{1}{2}\sigma'_X A\int_{z}^{\infty} dz\,\rho(r)}$$

Where

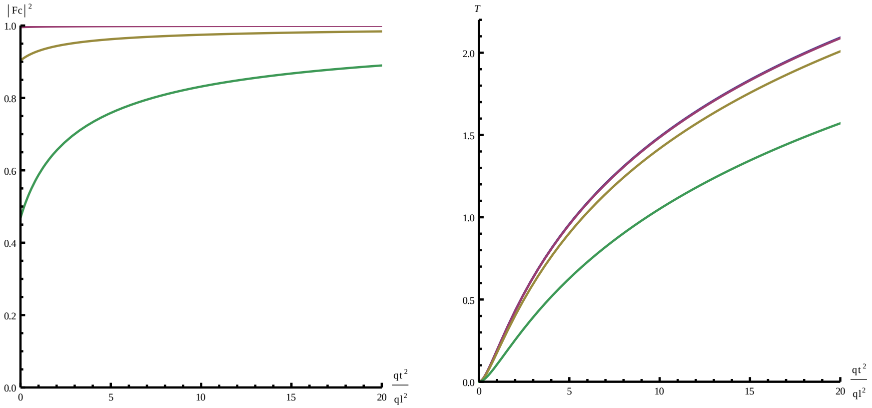
$\sigma' = \sigma_{tot}(1 - i\rho')$, ρ — normalized density,

$\chi_C(r) = \int_{-\infty}^{\infty} dz\,V(r)$, V — Coulomb energy ($\sim \alpha Z Z_B$)

Formfactor: Pointlike Nucleus

No charge distribution. No absorption.

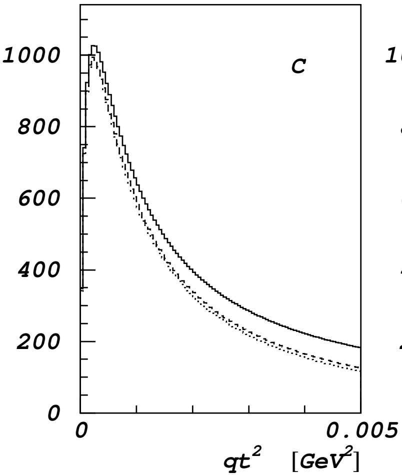
In that case $|F_C|^2 = f\left(\alpha Z Z_B,\, q_T^2/q_L^2\right)$
Shown are 4 curves: H, C, Cu, Pb ($Z = 1, 6, 29, 82$)



Formfactor: Uniform Solid Sphere R

Three curves: $F_C = 1$, finite nucleus size, plus absorption

Numerical calculations with old program



Production of the $K\pi$ system

Allowed spin/parity

$J^P = 0^+, 1^-, 2^+, 3^-, \dots$

For any production mechanism

$$\frac{d\sigma}{d\Omega} = |f(\Omega)|^2$$

$$f(\Omega) = \frac{1}{p_i} \sum_J (J + \tfrac{1}{2}) \langle \lambda_c \lambda_d | T^J | \lambda_a \lambda_b \rangle \, D^{J*}_{\lambda_a - \lambda_b, \lambda_c - \lambda_d}(\varphi, \theta, 0)$$

Assuming P-conservation and spin-0 beam, target, recoil

$A(-\lambda) = -A(\lambda)$ thus $A(0) = 0$

At small θ we have: $D^{J*}_{\lambda 0}(\Omega) \sim \theta^\lambda \sim q_T^\lambda$ thus $\frac{d\sigma}{dt} \sim t$

In Primakoff production

It seems that due to $\epsilon_{\mu\nu\lambda\rho}$ general expressions coinside with exact ones not only for K^*, but for any $K\pi$ system

Strong Production Cross Sections

Approximate cross section A-dependence for most processes is

$$\sigma \sim A^{2/3}$$

Incoherent production	Coherent $M = 0$ production	Coherent $M = 1$ production
$\frac{d\sigma}{dt} = C_S e^{-b_1 t}$	$\frac{d\sigma}{dt} = C_S e^{-b_1 A^{2/3} t}$	$\frac{d\sigma}{dt} = C_S t e^{-b_1 A^{2/3} t}$
$C_S \sim A^{2/3}$	$C_S \sim A^{4/3}$	$C_S \sim A^2$

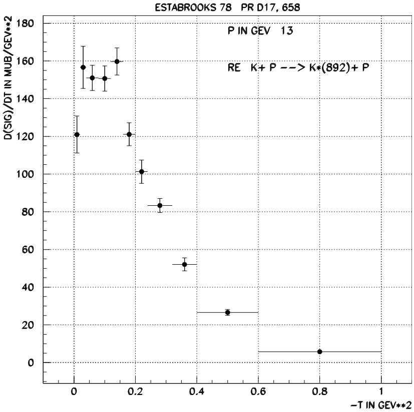
Background to Primakoff production lies in the small t region

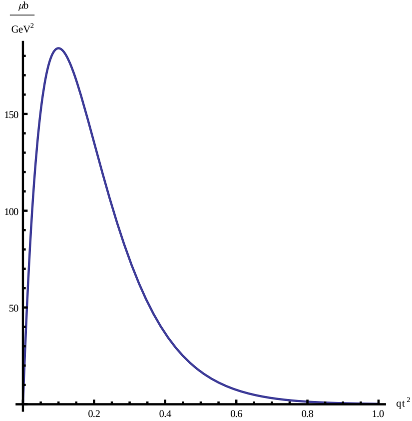
$$\text{background} \sim C_S$$

Experimental Data On $K^*(892)^+$ Production

13 GeV: 71 μb
30 GeV: 28 ± 5, 35 ± 5, 43 ± 5
50 GeV: 12 ± 3, 22 ± 2

$C_S\, t\, \exp(bt)$
$C_S = 5000$, $b = -10$
Equivalent to 50 μb

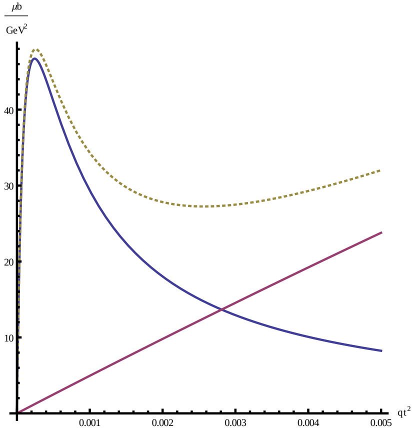




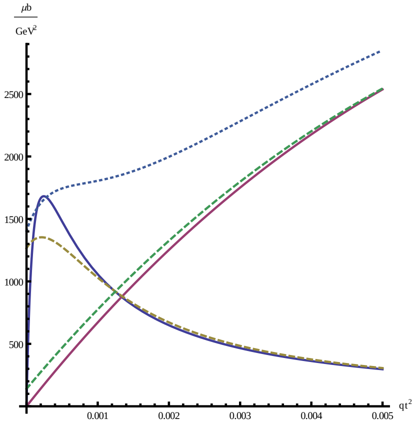
$K^*(892)^+$ Production Expectations

Assuming $F_C = 1$

$Z = 1,\ A = 1,\ b = -10\,\mathrm{GeV}^{-2}$
$C_S = 5000\,\mu\mathrm{b}/\mathrm{GeV}^4$



$Z = 6,\ A = 12,\ b = -70\,\mathrm{GeV}^{-2}$,
$C_S = 5000\,A^2\,\mu\mathrm{b}/\mathrm{GeV}^4$
Smearing: $\sigma_x = \sigma_y = 0.010\,\mathrm{GeV}$



Transferred Transverse Momentum

$d\sigma/dq_T^2$ peaks at $q_T^2 = q_L^2$

$q_L \approx 15.6\,\text{MeV}$ for $K^*(892)^+$

$q_L \approx 4.5\,\text{MeV}$ for $K\pi$ at the threshold

Transverse Momentum Resolution

$p_x = p\theta_x$

$\sigma[p_x] = p\sigma[\theta_x] + \theta_x\sigma[p]$

Due to momentum measurement

$$\sigma[p_x] = \theta_x\sigma[p] = \theta_x p\frac{\sigma[p]}{p} \lessapprox \frac{p^*}{\sqrt{2}}\frac{\sigma[p]}{p} = \frac{289\,\text{MeV}}{\sqrt{2}}\frac{\sigma[p]}{p} \approx \text{few MeV}$$

Due to multiple scattering

$$\sigma[p_x] = p\sigma[\theta_x] \approx p\frac{13.6\,\text{MeV}}{p}\sqrt{x/X_0} = 13.6\,\text{MeV} \times \sqrt{x/X_0}$$

Cross Sections And Luminosities

Input: $\Gamma[\mathrm{K}^*(892)^+ \to \mathrm{K}^+\gamma] = 50 \pm 5\,\mathrm{keV}$

Possible 10% X_0 targets

$$\mathcal{L} = \rho \mathrm{L N_A}/\mu \qquad \mathcal{L}_{\mathrm{C}} = 0.214\,\mathrm{b}^{-1} \qquad \mathcal{L}_{\mathrm{Cu}} = 0.0122\,\mathrm{b}^{-1}$$

Primakoff narrow K^* production assuming $|\mathrm{F_C}| = 1$

(A,Z)	max $\mathrm{q_T^2}$ [$\mathrm{GeV^2}$]	σ_{prim} [μb]	Prob[10%$\mathrm{X_0}$]
C	0.001	1.352	$0.29 \cdot 10^{-6}$
Cu	0.001	31.59	$0.39 \cdot 10^{-6}$

Note: 2 mm Cu $\approx$ 14% X_0

Events Expected

Primakoff K^{*+} for 10% X_0 C target with $q_T^2 < 0.001\,\mathrm{GeV}^2$

$0.5 \cdot 10^6$ K^+ per spill

1 day perfect accelerator running ($24 \times 60 \times 6 = 8640$ spills)

50% dead time

0.8 - formfactor and mass window selection effects

$$\mathrm{Rate} \approx 500 \times \mathrm{BR} \times \mathrm{Eff} \quad \frac{\mathrm{events}}{\mathrm{day}}$$

Other Primakoff production processes

$$\frac{\sigma[\mathrm{p} \to \Delta(1232)^+]}{\sigma[\mathrm{K}^+ \to \mathrm{K}^*(892)^+]} \approx 11 \qquad \frac{\sigma[\pi^+ \to \rho^+]}{\sigma[\mathrm{K}^+ \to \mathrm{K}^*(892)^+]} \approx 1.3$$

Outlook