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PHENOMENOLOGY OF THE CHIRAL ANOMALY

A Dissertation Presented

by

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Submitted to the Graduate School of the
University of Massachusetts Amherst in partial fulfillment
of the requirements for the degree of

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To My Parents

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ABSTRACT

PHENOMENOLOGY OF THE CHIRAL ANOMALY

FEBRUARY 1999

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In this thesis, we have explored the rich phenomenology of the chiral anomaly in order to further our understanding of this fascinating and unique aspect of quantum field theory. We studied a subset of anomalous processes viz., $\eta/\eta' \to \pi^+\pi^-\gamma$, $\gamma\gamma \to 3\pi$, and $\gamma \to 3\pi$ with the explicit purpose of extending the current status of calculations in this sector. For this, we applied the N/D formalism, which exploits the unitarity and analyticity of the scattering amplitude, in order to provide a plausible extension of one-loop results into resonance dominated regions of phase space. In particular, we were able to account for vector meson exchange effects by demanding that the phase shifts of the N/D function reflect those appearing in $\pi\pi$-scattering. We also compared our results with calculations based on vector dominance models, for which purpose we adopted the hidden symmetry approach.

We found that the N/D method is able to provide a smooth bridge between one-loop calculations in χPT at low energy and vector dominance inspired predictions for anomalous amplitudes. Finally, we listed some of the shortcomings of the approach and explored areas for further study of the phenomenology of the chiral anomaly.

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CHAPTER 1

INTRODUCTION

The chiral anomaly is a well established component of the strong interactions and other chiral quantum field theories. The anomaly manifests itself directly in the low energy interactions of the pseudoscalar mesons (π, K, η), including the well known $\pi^0 \rightarrow 2\gamma$ decay. Since the chiral anomaly originates from the chiral structure of QCD, quantitative tests of the anomaly can provide important information on the symmetries of the strong interactions. Unfortunately, while the theoretical origin and properties of the anomaly are well understood, its phenomenological implications for pseudoscalar meson physics have not been satisfactorily explored. This thesis attempts to partially redress this deficiency by studying a subset of anomalous reactions that are of significant experimental interest.

The thesis is organized as follows: The first three chapters provide the background information to motivate the original calculations of the later chapters. In the first chapter, we provide a general introduction to the anomaly, and present the Wess-Zumino-Witten Lagrangian which incorporates the anomalous interactions of the pseudoscalars at tree level in the chiral expansion. We then study the phenomenology of select anomalous processes, including $\gamma\pi \rightarrow \pi\pi$ and $\eta/\eta' \rightarrow \pi\pi\gamma$, which clearly indicate the need to go beyond tree level calculations in this sector. In chapter two, we present two different approaches to higher order corrections to anomalous processes viz., chiral perturbation theory and vector dominance models. We present the successful features and the limitations of each of these approaches. In chapter three, we introduce the N/D formalism as an alternative method to include higher order corrections, one that successfully incorporates the important phenomenological of chiral perturbation theory and vector dominance. The successful application of this method to the study of various anomalous decays forms the backbone of this thesis.

In chapter four, we apply these various formalisms to the $\eta/\eta' \to \pi\pi\gamma$ decays. Our goal is to extract, for the first time, the values of the $\eta - \eta'$ mixing angle and the pseudoscalar decay constants, F_8 and F_0, purely from the existing experimental data on the anomalous decays of the η/η'-system. To this end, we compare the energy dependence of the decay amplitudes in the physical region predicted by the three approaches mentioned above. We find that combining these predicted forms for the amplitudes with the data allows us to obtain the mixing parameters in a relatively model independent fashion.

Moving on to chapter five, we study the higher order corrections to a more complicated system - $\gamma\gamma \to 3\pi$. By assuming that the most important corrections to the amplitudes arise from elastic re-scattering in each of the two-pion channels in the final state, we are able to employ the N/D method and vector dominance to obtain predictions for the scattering amplitude. We then plot the predicted cross-sections and differential spectra for both $\gamma\gamma \to 3\pi^0$ as well as $\gamma\gamma \to \pi^+\pi^-\pi^0$, and compare our results with earlier published one-loop results. These cross-sections are of experimental interest for the DaΦne ϕ-factory in Frascati, Italy , where it is expected that these reactions will be observable.

Finally, in chapter six, we complete a comparative study of various model-dependent predictions for the anomalous $\gamma\pi \to \pi\pi$ process. We add two new features to earlier analyses. First, we extend the previous calculations up to and beyond the region of the ρ-peak, where experimental data can be expected to be quite sensitive to properties of the ρ-meson. Second, we compare the predictions of two different N/D forms, one of which is introduced in this work. Interestingly, we find that the flexibility in choosing the form of the appropriate amplitude is not reflected in the final results which closely match each other in the physical region.

We conclude the thesis by summarizing the important aspects of the theory and calculations presented in the thesis. We also point out some limitations of the N/D formalism and suggest various ways to improve its predictions and its range of applicability.

1.1 Symmetry Breaking and Anomalies

The phenomena of symmetries and invariance principles play a pivotal role in our current understanding of the fundamental interactions in nature. It is a remarkable fact that all the known fundamental forces can be described within the framework of gauge field theories, wherein an intimate link is forged between symmetry principles and the dynamical content of the theory. The requirement of local gauge invariance necessitates the presence of gauge fields which mediate the interactions of the various particles in the theory. For example, the $U(1)_V$ invariance of the QED lagrangian introduces the photon as the mediator of the electromagnetic force. Invariances under global symmetry transformations, though less restrictive, also provide important constraints on particle dynamics. Global $U(1)$ invariance preserves charge or particle number conservation within the theory.

Nature, however, seems not to have chosen the most symmetric path to development. Rather, one finds phenomenologically that particle interactions proceed through a maze of broken symmetries. One classic example of the profound consequences of such broken symmetries arises from CP-violation. While CP-symmetry was historically proposed as a possible cure for the breakdown of parity in the weak interactions, the deeper significance of CP-violation in nature arose from a study of the early universe. A CP-symmetric early universe would have contained an equal number of particles and antiparticles. Furthermore, it is expected that these particle-antiparticle pairs would have annihilated into photons, inundating the universe with light. Clearly, the building up of atoms and the elements (and life) could not have proceeded in such a world. Similar, though somewhat less profound, effects arise from other discrete and continuous symmetries when they are broken. The confrontation of theory with experiment in the fundamental interactions can therefore be restated as the interplay between invariance principles and broken symmetries.

The starting point for calculations in field theory is the classical lagrangian or classical action which is built up from the underlying symmetries of the theory. Symmetry breaking mechanisms at the level of the lagrangian can generally be classified into three types [1]:

(1) explicit, where the lagrangian itself contains terms that violate the symmetry. The presence of mass terms in a chiral theory explicitly breaks the chiral symmetry. The isospin symmetry of the strong interactions is also explicitly broken by the u-d quark mass difference.

(2) spontaneous, where the lagrangian is invariant under the symmetry transforms but the ground state, out of which the particle spectrum is built up, is not. A classic example of this is the ferromagnet. While the underlying lagrangian is rotationally invariant, the ground state consists of all dipoles lined up in some fixed but arbitrary direction.

(3) quantum mechanical or anomalous, where invariance of the action at the classical level cannot be preserved when quantum corrections are taken into account. The phenomenological consequences of anomalies in the arena of the strong interactions is the focus of this work. However, anomalies arise in a number of different situtations not involving quantum field theory. We refer the reader to the literature for excellent reviews of anomalies in ordinary quantum mechanics and other areas [2].

To describe the phenomenon of anomalies, one works within the framework of perturbative field theory [3]. For a set of fields ϕ_i, the lagrangian $\mathcal{L}$ is generally a function of the fields and their derivatives.

$$\mathcal{L} = \mathcal{L}(\phi_i, \partial_\mu \phi_i) \tag{1.1}$$

Invariances or symmetries of a classical theory are tested through the lagrangian. Consider an infinitesimal transformation of the fields ϕ_i

$$\phi_i \rightarrow \phi_i' = \phi_i + \epsilon(x) f_i(\phi_i) \tag{1.2}$$

If the lagrangian is invariant under the transformation of the fields, then the equations of motions and the physics will remain unchanged. In such cases, one can construct conserved currents and charges by defining the current through

$$J^\mu(x) = \frac{\partial}{\partial(\partial_\mu \epsilon(x))} \mathcal{L}(\phi_i', \partial_\mu \phi_i') \tag{1.3}$$

Using the Euler-Lagrange equations, and requiring the invariance of the lagrangian under the transformation yields

$$\partial_\mu J^\mu(x) = \frac{\partial}{\partial \epsilon(x)} \mathcal{L} = 0 \tag{1.4}$$

i.e., the current is conserved. One can also define a time-independent conserved charge through the zeroth component of the current four-vector.

$$Q = \int d^3x \; J^0(x) \tag{1.5}$$

The presence of these conserved currents and charges are the essence of Noether's theorem viz., the invariances of a langrangian under continuous field transformations lead to conserved quantities and conservation laws.

Invariances in quantum theory, however, are tested through matrix elements or the path integral. An anomaly is said to occur when a symmetry of the classical action is not shared by the path integral i.e., by the full quantum theory. The classically conserved currents pick up anomalous contributions and the current divergences are no longer zero. Let us study this briefly for the theory of a Dirac fermion interacting with an external electromagnetic source. We shall only state the main results of the calculation. The interested reader can find the details of the derivation in the original paper by K. Fujikawa or in some well known texts [1, 4].

The classical action is given by

$$\begin{aligned} S &= \int d^4x \, \overline{\psi}(i\not{D} + im)\psi \\ D_\mu &= \partial_\mu + ieA_\mu \end{aligned} \tag{1.6}$$

Requiring the invariance of the action S under global phase and chiral transformations

$$\begin{aligned} \psi &\rightarrow e^{i\alpha}\psi \\ \psi &\rightarrow e^{i\beta\gamma_5}\psi \end{aligned} \tag{1.7}$$

leads to the following vector and axial-vector currents

$$\begin{aligned} V_\mu(x) &= i\overline{\psi}\gamma_\mu\psi \qquad \partial_\mu V^\mu(x) = 0 \\ A_\mu(x) &= i\overline{\psi}\gamma_\mu\gamma_5\psi \qquad \partial_\mu A^\mu(x) = -2m\overline{\psi}\gamma_5\psi \end{aligned} \tag{1.8}$$

which are clearly conserved in the massless limit. The last result was obtained by briefly considering $\beta = \beta(x)$ and setting the change in S to zero.

$$\delta S = 0 = \int d^4x \beta(x)[-\partial_\mu(i\overline{\psi}\gamma_\mu\gamma_5\psi) - 2m\overline{\psi}\gamma_5\psi] \tag{1.9}$$

In the quantum theory, however, it is the path integral that is relevant, and this is given in Euclidean space as

$$I = e^{-Z[A]} = \int [D\psi^\dagger][D\psi]\; e^{-S[\psi,\ \psi^\dagger,\ A]} \tag{1.10}$$

Under the chiral tranformation, the action changes as before (for x-dependent β), but now the fermionic measure also changes,

$$[D\psi^\dagger][D\psi] \rightarrow J[\beta][D\psi^\dagger][D\psi] \tag{1.11}$$

Following a lengthy calculation, one obtains

$$J[\beta] = e^{-\frac{1}{8\pi^2}\int d^4x\beta(x)F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x)} \tag{1.12}$$

where $F_{\mu\nu}$ is the electromagnetic field tensor and $\tilde{F}^{\mu\nu}$ is the dual tensor given by

$$\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta} \tag{1.13}$$

Hence, requiring $\delta I = 0$ under the chiral transformation yields

$$\partial_\mu A^\mu(x) = -2m\bar{\psi}\gamma_5\psi + \frac{ie^2}{8\pi^2}F_{\mu\nu}(x)\tilde{F}^{\mu\nu}(x) \tag{1.14}$$

Clearly, employing the path integral introduces a new ingredient in the recipe for conserved currents viz., the fermionic determinant, which provides an anomalous contribution to a classical current divergence. We shall later remark on the specific result obtained.

Anomalies can arise in either global or local gauge symmetries of a field theory. Global anomalies contribute to physical processes. For example, the global $U(1)$ axial anomaly in QCD prevents the η' from being a Goldstone boson. Local anomalies, however, are disastrous for gauge theories. In such theories, the gauge fields are coupled to anomalous currents which are not conserved. Such theories cannot be consistently quantized and are non-renormalizable. One such example is the $U(1)$ hypercharge anomaly in the Standard Model. Hypercharge assignments to the fermions in the Standard Model are carefully arranged to cancel the anomaly. This ensures the renormalizability of the electroweak theory [1]. A similar cancellation mechanism has to be employed in all gauge theories coupled to fermions.

We are interested in the case where anomalies arise from the violation of global chiral symmetries. In general, chiral anomalies occur in theories involving chiral fermions. Such theories contain both, vector as well as axial-vector currents arising from chiral invariance of the lagrangian. As we will show later, quantum effects deny the possibility of the simultaneous conservation of both sets of currents. Since conservation of the vector current is intimately linked to gauge invariance, it is imposed on the quantum theory. Consequently, anomalous terms appear in the divergences of the axial currents. These lead to measurable phenomena such as the well known decay of the neutral pion into two photons. We shall discuss this decay next in order to demonstrate how the chiral anomaly manifests itself within a 1-loop calculation, as well as to introduce the famous "triangle" diagram. This will also provide an natural introduction to effective lagrangian techniques in QCD which incorporate the effect of the chiral anomaly in a systematic fashion.

1.2 PCAC and Ward Identities in $\pi^0 \to \gamma\gamma$

We begin with a historical note. The idea of quantum mechanical symmetry violation was put forth by Niels Bohr on two separate occasions. Resisting Einstein's concept of the photon as a quantum, Bohr suggested that energy conservation was violated in the photoelectric effect. He later made a similar suggestion for β-decay which turned out to be incorrect when the neutrino was discovered. However, the