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Some Rare Decays of Light Mesons[1]

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Abstract

I give a review of several decay modes of light mesons (π, η, η',..) which are rare or forbidden in the standard model of particle physics. In particular, the weak and electromagnetic decays of the π^0, $\pi^\pm$, η, and η' mesons are discussed in some detail. Some new results on K decays are also discussed. The sensitivity of these decays to parameters of the standard model as well as to non-standard physics is investigated.

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It has been increasingly difficult to find experimental indications of an eventual breakdown of the standard $SU_3 \times SU_2 \times U_1$ model of the strong and electroweak interactions.

From the very successful first years of LEP operation, we know that there are no more than 3 fermion generations with light neutrinos, the minimal model Higgs particle is heavier than around 60 GeV, and the hadronic decays of the Z^0 are perfectly consistent with QCD. Still, we are stuck with the 20-odd free parameters of the standard model that lack an explanation in a grander theory. With the proton unwilling to decay even in 10^{33} years, the previously so attractive idea of a Grand Unification of strong and electroweak interactions has lost some of its power (although some of the LEP findings combined with other data may perhaps point to a supersymmetric Grand Unification [1]). The tremendous technical difficulties encountered in superstring theories seem to imply that we have to wait a long time until we get some input into low energy phenomenology from there. So, the question of how proceed is now back in the hands of the experimentalists. The obvious road of going to higher energies will undoubtedly be pursued, but the time scale for projects such as the SSC or LHC is on the order of a decade, so we should also see what can be achieved by other roads to new physics. One new, very promising such road is that of astroparticles where, indeed, the solar neutrino deficit and the dark matter problem may be the first indications that there exist fields and interactions beyond the Standard Model. Another promising road, which we will follow here, is that of precision measurements and of rare and forbidden decays. In this talk I will concentrate on weak and electromagnetic decays that are allowed within the standard model, although they are in several cases expected to be highly suppressed. Besides being interesting to study in their own right, providing challenges for the careful experimentalist, they are in many cases important background processes for more exotic decays beyond the minimal standard model. With the new powerful experimental facilities providing much more intense beams of particles than ever before, it is time to go back to age-old questions about the structure, and also, fine structure of the electroweak and strong interactions to a new level of precision.

The study of the pion has of course been of utmost importance for the evolution of modern particle physics. We may only recall the importance of the leptonic decays of the charged pions for establishing the helicity suppression that follows from the $V-A$, and therefore chiral, structure of the weak interactions. Of equal importance was the calculation of the decay rate of $\pi^0 \to \gamma\gamma$ through the axial vector anomaly [2, 3].

We write

$$\mathcal{M}(\pi^0 \to \gamma\gamma) = ie^2\epsilon_\mu(k_1)\epsilon_\nu(k_2)T^{\mu\nu}, \qquad (1)$$

with

$$T_{\mu\nu} = \int d^4x e^{ik_1\cdot x} < 0|T[J_\mu(x)J_\nu(0)]|\pi^0 >, \qquad (2)$$

where the electromagnetic current is

$$J_\mu = \sum_f Q_f \bar{\Psi}_f \gamma_\mu \Psi_f, \qquad (3)$$

extending the sum over all charged fermion fields (Q_f is the charge of the fermion in units of the proton charge). Now, gauge invariance, parity conservation and Bose symmetry restricts $T_{\mu\nu}$ to be of the form

$$T_{\mu\nu} = iF(0,0)\epsilon_{\mu\nu\rho\sigma}k_1^\rho k_2^\sigma, \qquad (4)$$

where for future use we have introduced the electromagnetic form factor $F(k_1^2, k_2^2)$, which in the case of real photons is evaluated at $k_1^2 = k_2^2 = 0$. The decay width for $\pi^0 \to \gamma\gamma$ is then:

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\pi\alpha^2 m_\pi^3 |F(0,0)|^2}{4}. \tag{5}$$

It remains to calculate $F(0,0)$. Writing $|\pi^0> = |u\bar{u} - d\bar{d}> /\sqrt{2}$, and using the PCAC condition for the $\pi^0 q\gamma_5\bar{q}$ couplings $g_{u\bar{u}} = \sqrt{2}m_u/f_\pi$, $g_{d\bar{d}} = -\sqrt{2}m_d/f_\pi$ (here f_π is the charged pion decay constant, $f_\pi = 132$ MeV) one finds [3]

$$F(0,0) = \frac{\sqrt{2}N_c(Q_u^2 - Q_d^2)}{4\pi^2 f_\pi}, \tag{6}$$

where N_c is the number of colours. Inserting $N_c = 3$, $Q_u = 2/3$, $Q_d = -1/3$, the result is

$$F(0,0) = \frac{\sqrt{2}}{4\pi^2 f_\pi}, \tag{7}$$

implying

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{\alpha^2 m_\pi^3}{32\pi^3 f_\pi^2} \sim 7.6 \text{ eV}, \tag{8}$$

which compares favourably with the experimental value [4] $\Gamma_{exp}(\pi^0 \to \gamma\gamma) = (7.24 \pm 0.08 \pm 0.12)$ eV. Indeed, the success of this calculation is still today one of the most convincing demonstrations of the need for three colours in the fundamental representation of the strong interaction gauge group. Note that the calculation of this rate depends on the assumption that the matrix element of the divergence of axial vector to a two-photon state vanishes, and therefore the whole contribution of the anomaly is picked up by the pseudoscalar current. In the limit of a massless pion this is exact due to a theorem by Veltman and Sutherland [5], and the corrections stemming from the extrapolation to the physical pion mass have been estimated to be small [6]. Indeed, calculating in a nucleon loop model containing pseudoscalar couplings, Steinberger obtained a reasonable result for the $\pi^0 \to \gamma\gamma$ decay rate already in 1949 [7].

One may ask about the robustness of the anomaly result in another fashion. As we know, anomalies in gauge theories usually spell disaster since they violate chiral Ward identities which are crucial for a consistent renormalization. In the case of $\pi^0 \to \gamma\gamma$, however, we are talking about an effective theory at low energies, and the axial current coupling to the pion is not a gauge current. In elementary treatments of the anomaly it is often stressed that it appears perturbatively as an ambiguity of defining finite parts of the triangle diagrams due to the linear divergence of individually contributing graphs. In this view, it is not obvious that the anomaly survives with full strength even if it arises from a convergent set of diagrams that only become divergent due to a contraction of a heavy propagating field to a point. However, this is indeed so, and a heuristic argument was given by Dolgov and Zakharov [8] who showed that it is essentially a low-energy phenomenon unrelated to the eventual divergence or non-divergence of the triangle graphs, as long as the relevant mass scale is below the mass scale defining the cutoff of the theory. A very clear illustration of this has appeared in the calculation of photino annihilation into a photon pair [9]. In supersymmetric solutions of the dark matter problem, the cold dark matter halo is composed of the lightest supersymmetric particles (LSP) (e.g., the photino; in general the LSP is a mixture of several components). Since they move at non-relativistic speeds in the halo, and since they are Majorana particles, it turns out that they have to annihilate from an initial pseudoscalar state so that the annihilation process is very similar to $\pi^0 \to \gamma\gamma$, i.e., it is given by the anomaly. The nice thing is, however, that in this case we have a complete model for the anomaly since in the underlying renormalizable supersymmetric theory the cutoff is given by propagating squarks and sleptons. Therefore, the whole process can be calculated as a box diagram which is completely convergent. Indeed, as shown in [9], the pointlike (anomaly) approximation reproduces the full result to a remarkable accuracy for an initial state mass all the way up to the cutoff mass. In this sense, the anomaly result for radiative decays and annihilations is very stable.

Amazingly, there exists another, *a priori* completely unrelated method of obtaining a good value for the $\pi^0 \to \gamma\gamma$ decay rate. It is based on the Vector Meson Dominance (VMD) assumption (for a review, see [10]). According to this model, the $\pi^0 \to \gamma\gamma$ decay is dominated by the off-shell $\pi\rho\gamma$ and $\pi\omega\gamma$ transitions followed by a vector meson - γ transition. Writing for $V = \rho, \gamma$

$$< 0|J_\mu^{em}|V >= \epsilon_\mu m_V^2 f_V, \tag{9}$$

$$\mathcal{M}(V \to \pi^0\gamma) = e\widetilde{F}^{\mu\nu} P_\mu \epsilon_\nu f_{V\pi\gamma}, \tag{10}$$

one gets, using the quark model relations $f_\omega = f_\rho/3$, $f_{\omega\pi\gamma} = 3f_{\rho\pi\gamma}$, the prediction

$$\Gamma(\pi^0 \to \gamma\gamma) = \frac{18m_\pi^3\Gamma(\rho \to e^+e^-)\Gamma(\rho \to \pi\gamma)}{\alpha m_\rho^4(1 - m_\pi^2/m_\rho^2)^3} \sim (8.4 \pm 1.3) \text{ eV}, \tag{11}$$

which is again in excellent agreement with the experimental value. The corrections to this relation come mainly from continuum contributions. These can be estimated using QCD sum rules and have been found to be small [11].

The fact that the perturbative triangle diagram describes the $\pi^0 \to \gamma\gamma$ so well is related to the non-renormalization property of the anomaly [12]. Above 1 GeV perturbative QCD already begins to become successful, so there is only an intermediate mass region around the ρ mass where calculations become difficult. VMD on the other hand has an non-perturbative basis in dispersion relations and may therefore be applied also in the intermediate region. Indeed, one expects the pion two-photon form factor $F(s_1, s_2)$ to extrapolate from vector meson dominance at low s_1 and s_1 to the predicted QCD behaviour at large momentum transfers. To verify this experimentally, one has to study processes like the single Dalitz decay $\pi^0 \to e^+e^-\gamma$, the double Dalitz decay $\pi^0 \to e^+e^-e^+e^-$, the rare decay $\pi^0 \to e^+e^-$, or the pion production processes $e^+e^- \to e^+e^-\pi^0$, $e^+e^- \to \pi^0\gamma$ or $Z^0 \to \pi^0\gamma$.

An amusing electromagnetic decay process of the π^0 is $\pi^0 \to$ positronium $+ \gamma$. This occurs as a competing process to the Dalitz decay $\pi^0 \to e^+e^-\gamma$ at the lowest possible invariant mass of the lepton pair (in fact, below the e^+e^- threshold, since the Coulomb binding energy is negative). Since the virtual photon carries $J^{PC} = 1^{--}$, only the n^3S_1 will give a sizable contribution (since the positronium atom is nonrelativistic, the wave function at the origin of the n^3D_1 states vanishes to lowest order as $(v/c)^2$). The calculation of the rate, which is a nice exercise in QED and the nonrelativistic bound state formalism, was first carried out by Nemenov [13]. He found

$$\frac{\Gamma(\pi^0 \to \text{positronium} + \gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = 32\pi\alpha(1 - \frac{4m_e^2}{m_\pi^2}) \sum_{n=1}^{\infty} \frac{|\Psi_n S(0)|^2}{4m_e^2} \sim 0.6\alpha^4 \sim 1.7 \cdot 10^{-9}, \quad (12)$$

where the numerical value $\sum_{n=1}^{\infty} 1/n^3 = \zeta(3) = 1.202\ldots$ has been inserted. The order α radiative corrections coming from the vacuum polarization of the virtual photon and transverse photon exchange in the n^3S_1 bound states have also been computed [14] with the result

$$\Gamma = \Gamma_0(1 - \frac{26\alpha}{9\pi}). \quad (13)$$

There have recently been measurements of this process in $pC \to \pi^0 + X$ reactions at 70 GeV [15]. The published set of data gives a value of $(1.8\pm0.3)\cdot 10^{-9}$ for the branching ratio, in good agreement with the QED prediction (we note that at $s = 4m_e^2$ strong interaction effects in $F(s,0)$ should be completely negligible).

We now come to the question of the general form of the two-photon form factor $F(s_1, s_2)$ for off-shell photons. From now on we normalize all processes to $\pi^0 \to \gamma\gamma$, so that we put $F(0,0) = 1$. The differential distribution for the Dalitz decay $\pi^0 \to e^+e^-\gamma$ is then

$$\frac{1}{\Gamma(\pi^0 \to \gamma\gamma)} \frac{d\Gamma(\pi^0 \to e^+e^-\gamma)}{ds} = \frac{2\alpha}{3\pi s}(1 - \frac{s}{m_\pi^2})^3(1 + \frac{2m_e^2}{s})\sqrt{1 - \frac{4m_e^2}{s}}|F(s,0)|^2, \quad (14)$$

where s is the invariant mass squared of the lepton pair. The most direct prediction for the behaviour of the form factor $F(s,0)$ comes from the VMD model:

$$F^{VMD}(s,0) = \frac{m_\rho^2}{m_\rho^2 - s}, \quad (15)$$

where m_ω has been put equal to m_ρ. It is customary to introduce the dimensionless variable $x = s/m_\pi$, and to parametrize the Dalitz form factor at low x according to $F(x) = F(s/m_\pi^2, 0) = 1 + ax$. The prediction from VMD is thus $a = m_\pi^2/m_\rho^2 \sim 0.03$. With such a small slope, the influence of the form factor on the total branching ratio for $\pi^0 \to e^+e^-\gamma$ is negligible. The branching ratio for constant $F(x) = 1$ is given by

$$\frac{\Gamma(\pi^0 \to e^+e^-\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{4\alpha}{3\pi}\left(\log(m_\pi/m_e) - \frac{7}{4}\right) \sim 1.19 \cdot 10^{-2}, \quad (16)$$

which is in good agreement with the experimental value [16] $(1.20 \pm 0.03) \cdot 10^{-2}$.

An interesting question is what happens to the triangle diagram calculation for an off-shell photon. The general form factor for both photons and the pion off-shell has been given in [17]. For the Dalitz decay considered here the result is

$$F^{loop}(s,0) = \frac{m_\pi^2}{m_\pi^2 - s}\left(1 - \frac{\arcsin^2(\sqrt{s}/2M)}{\arcsin^2(m_\pi/2M)}\right), \quad (17)$$

where M is the mass of the quark in the triangle loop diagram. The small s expansion of this form factor is, in the limit $m_\pi \to 0$,

$$F^{loop}(s,0) \sim 1 + \frac{s}{12M^2} + \ldots . \quad (18)$$

It has been noticed [18, 19] that this prediction can be made consistent with the VMD prediction by the choice $M \sim m_\rho/2\sqrt{3} \sim 150-200$ MeV. This means effectively replacing the current quark masses of the PCAC calculation by constituent quark masses. Here higher order chiral and QCD corrections will most certainly be important, however, so this "Q^2 duality" should not be taken too seriously. The appearance of the constituent quark mass in this context needs justification by a more reliable calculation. Recently, this has been attempted [20] using the effective Nambu-Jona-Lasinio theory, where indeed this assumption is shown to be justified if the strong interaction cutoff is taken to be around 0.5 GeV.

For values of s much larger than 1 GeV^2, the $\pi^0\gamma^*\gamma$ transition form factor should be calculable in QCD. This has been done [21, 22], and one finds asymptotically

$$F^{QCD}(s \to \infty, 0) \sim \frac{3}{2s}. \tag{19}$$

The approach to the asymptotic value is logarithmic and in fact very slow [22]. It is instructive to note that all three of these models, VMD, quark loop and QCD predict a form factor falling like $1/s$ (modulo logarithmic corrections) for $s >> 1$ GeV^2. In particular, this means a very small prediction, $\sim 10^{-10}$ or so, for the branching ratio of $Z^0 \to \pi^0\gamma$ which also proceeds through the vector current [21, 24, 25]. A similar prediction emerges from the non-relativistic bound state model [26, 27], if it is extrapolated down to the pion mass. This seemingly solid result was questioned by Jacob and Wu [28] who predicted a much higher value, but indeed, in accordance with our discussion, the first experimental upper limits on this decay from the ALEPH [29] and OPAL [30] collaborations at CERN seem to rule out such a high value.

The experimental situation regarding the decay $\pi^0 \to e^+e^-\gamma$ has become more conclusive recently. There was an experiment (at Saclay, [32]) a couple of years ago reporting a *negative* value for the slope parameter, $a \sim -0.11 \pm 0.03 \pm 0.08$. This is of course quite difficult to reconcile with the theoretical prediction, and the most recent measurement of the Dalitz form factor [33] indeed measures a positive slope $a \sim 0.025 \pm 0.014 \pm 0.026$, consistent with the VMD prediction within the large errors. In fact, the best results on the transition form factor have recently come from measurements in the space-like region. Results from the CELLO collaboration at DESY [34] on light meson production in $\gamma\gamma$ and $\gamma^*\gamma$ processes give a from factor slope which is in excellent agreement with vector meson dominance both for π^0, η and η'.

The QED radiative corrections to $\pi^0 \to e^+e^-\gamma$ seem to be well understood [35]. The total correction amounts as is usual for a QED correction only to a per cent or so, but the corrections to the differential decay rate and therefore to the slope parameter are more sizable [35] and in fact essential to include when extracting an experimental value.

The vector model dominance behaviour of the form factor is better tested in η and η' decays, where a larger range of s is kinematically available. Indeed, the existing, although scarce, data support the VMD picture for these decays (for a thorough review, see the article by Landsberg [36]). Also in the decays $\omega \to \pi^0\mu^+\mu^-$ and η, $\eta' \to \pi^+\pi^-\gamma$ there is experimental support for ρ dominance. In some of these experiments there are problems with the position of the ρ pole, which sometimes seems displaced to a higher value. This could be due to an interference with non-resonant contributions.

We now turn to an interesting decay of higher order in α, namely the rare decay of a neutral pseudoscalar meson into a lepton pair, e.g., $\pi^0 \to e^+e^-$, $\eta, \eta' \to \mu^+\mu^-$ or e^+e^-. These decays are highly suppressed in the standard model since they are of order α^4 and, in addition, suppressed by a chirality factor $\sim m_\ell^2$. Therefore, it has been realised by many authors that this is a channel where more exotic interactions could have a chance to show up (e.g., [38-46]).

The electromagnetic contribution to this decay is given by [44] (we concentrate first on the $\pi^0 \to e^+e^-$ decay)

$$\frac{\Gamma^{QED}(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} = 2\sqrt{1 - \frac{4m_e^2}{m_\pi^2}} \left(\frac{\alpha m_e}{\pi m_\pi}\right)^2 |R|^2, \tag{20}$$

where the dominant contribution of the imaginary part of R is given by the real two-photon intermediate state and is therefore model independent. It is

$$Im\ R = \frac{\pi}{2v} \log\left(\frac{1+v}{1-v}\right), \tag{21}$$

with $v = \sqrt{1 - 4m_\ell^2/m_\pi^2}$. The real part is, on the other hand, model dependent and is given by an integral over the two virtual photon form factor:

$$Re\ R = \frac{i}{\pi^2 m_\pi^2} \int d^4k \frac{2(P^2k^2 - (Pk)^2)F(k^2, (P-k)^2)}{k^2(P-k)^2[(p_e - k)^2 - m_\ell^2]}. \tag{22}$$

Here P is the four momentum of the pion, p_e that of the electron, and F is the two-photon form factor introduced before (normalized so that $F(0,0) = 1$).

There have been several attempts to calculate $Re\ R$ using various models for the form factor (among the more recent are [17, 44, 19, 47, 48]). It may at first seem hopeless to extract an unambiguous prediction for the electromagnetic $\pi^0 \to e^+e^-$ decay rate due to the appearance of the form factor F (especially since the real part can be shown to be logarithmically divergent in the point-like limit). However, as was shown in [18], in the particular case of π^0 decay one may use the fact that there is a hierarchy of mass scales, $m_e^2 \ll m_\pi^2 \ll \Lambda^2$ (Λ is a typical hadronic mass scale), to extract a firm prediction:

$$Re\ R = \log^2(m_\pi/m_e) - 3\log(\Lambda/m_e) + \text{const.} \tag{23}$$

The theoretical prediction for the $\pi^0 \to e^+e^-$ decay is thus

$$\frac{\Gamma^{QED}(\pi^0 \to e^+e^-)}{\Gamma(\pi^0 \to \gamma\gamma)} = (0.65 \pm 0.15) \cdot 10^{-7}, \tag{24}$$

where the error estimate is based on the spread of the result in the various models for the form factor.

There has recently appeared an interesting new calculation of this decay in chiral perturbation theory [49]. The $\eta\gamma\gamma$ and $\pi^0\gamma\gamma$ vertices arise from the Wess-Zumino-Witten term [50]

$$\mathcal{L}_{WZW} = \frac{\alpha}{4\pi f_\pi} \epsilon_{\mu\nu\lambda\sigma} F^{\mu\nu} F^{\lambda\sigma} \left(\pi^0/\sqrt{2} + \eta/\sqrt{6}\right) + \ldots\ldots \quad , \tag{25}$$

As we said above, the real part diverges in an effective theory like this and requires a local counterterm

$$\mathcal{L}_{\text{c.t.}} = \frac{3i\alpha^2}{32\pi^2}\bar{\ell}\gamma^\mu\gamma_5\ell[\chi_1 Tr(Q^2\Sigma^\dagger\partial_\mu\Sigma - Q^2\partial_\mu\Sigma^\dagger\Sigma) + \chi_2 Tr(Q\Sigma^\dagger Q\partial_\mu\Sigma - Q\partial_\mu\Sigma^\dagger Q\Sigma)] \ , \tag{26}$$

where $\ell = e$ or μ, Q is the electromagnetic charge matrix

$$Q = \begin{pmatrix} 2/3 & 0 & 0 \\ 0 & -1/3 & 0 \\ 0 & 0 & -1/3 \end{pmatrix} . \tag{27}$$

The field $\Sigma = \exp(i2M/f_\pi)$ is the exponentiation of the goldstone boson matrix M where

$$M = \begin{pmatrix} \pi^0/\sqrt{2}+\eta/\sqrt{6} & \pi^+ & K^+ \\ \pi^- & -\pi^0/\sqrt{2}+\eta/\sqrt{6} & \overline{K^0} \\ K^- & K^0 & -2\eta/\sqrt{6} \end{pmatrix} . \tag{28}$$

The coefficients χ_1 and χ_2 are renormalization scheme dependent and subtraction scheme dependent; the authors of [49] use dimensional regularization with $\overline{MS}$ (the gamma matrix algebra is performed in 4 dimensions) and choose the subtraction point to be $\Lambda = 1$ GeV. They find

$$ReR = \frac{1}{4}[\chi_1(\Lambda) + \chi_2(\Lambda) + 11 - 6\log(\frac{m_\mu^2}{\Lambda^2}) + 2\xi^2 - 4\xi^4 + 4\xi^2\log(4\xi^2) + 8\xi^4\log(4\xi^2)$$
$$-4\int_0^1 dy[3 + \frac{2(\xi^2-1)\sqrt{y}}{\sqrt{y+\xi^{-2}(1-y)}}]\lambda_+^2\log|\lambda_+| - 4\int_0^1 dy[3 - \frac{2(\xi^2-1)\sqrt{y}}{\sqrt{y+\xi^{-2}(1-y)}}]\lambda_-^2\log|\lambda_-|]. \tag{29}$$

Here, $\xi^2 = (1-v^2)^{-1}$ and $\lambda_\pm = \sqrt{y\xi^2} \pm \sqrt{y\xi^2 + (1-y)}$.

The real part of the amplitude is numerically in good agreement with the previous calculations mentioned above, which introduced a form factor for the $\eta\gamma\gamma$ vertex.

The branching fraction for $\eta \to \mu^+\mu^-$ has recently been remeasured at Saturne [51], where the value $\text{Br}(\eta \to \mu^+\mu^-) = (5 \pm 1) \times 10^{-6}$, is reported. This fixes the sum of the counterterms $-40 < \chi_1(\Lambda) + \chi_2(\Lambda) < -13$. The insensitivity of the real part of the amplitude to the specific model for the form factor discussed above reflects itself in this calculation through the fact that the rate is relatively insensitive to the precise value of the counterterms. This is because the one loop amplitude is infrared divergent as $m_\ell \to 0$ (the term proportional to $\log^2(m_P/m_\ell)$ in Eq. (23)) and so dominates the contribution from the counterterm [49].

Having fixed the sum of counterterms $\chi_1(\Lambda) + \chi_2(\Lambda)$ one may now predict the rates for $\eta \to e^+e^-$ and $\pi^0 \to e^+e^-$ (χ_1 and χ_2 are the same for the cases $l = e$ and $l = \mu$, because both the e and μ masses are small compared with the chiral symmetry breaking scale). One finds [49]

$$\text{Br}(\pi^0 \to e^+e^-) = 7 \pm 1 \times 10^{-8} \tag{30}$$
$$\text{Br}(\eta \to e^+e^-) = 5 \pm 1 \times 10^{-9} \tag{31}$$

For $\pi^0 \to e^+e^-$, this is in agreement with the "theorem" mentioned above.

The experimental situation has long been confused, with several experiments giving (although with large errors) values 2 to 3 times larger than the QED prediction [52, 53, 54]. However, recently a more sensitive PSI experiment [55] has failed to see the decay putting an upper limit of $1.3 \cdot 10^{-7}$ at 90% confidence level. It is imperative that experimenters continue the search for this decay which constitutes one of the few cases where there is still a large mismatch between the accuracy of theoretical predictions and that of experimental measurements.

Even if the QED prediction will eventually be confirmed, the $\pi^0 \to e^+e^-$ decay will still play a role of limiting various exotic couplings. As an example, the so-called "variant axion" [56] that was conjectured to explain the electron-positron peaks seen at the Darmstadt heavy ion experiments [57] could be excluded since it would, among other things, give too large a rate for the $\pi^0 \to e^+e^-$ decay [58, 59, 44].

The question of the radiative corrections to $\pi^0 \to e^+e^-$ has not yet been resolved satisfactorily. In the limit of a pointlike πee coupling, the calculation is straightforward although there are some subtleties related to renormalization. It was performed in [60], with the result

$$\frac{\Gamma^{\text{rad}}}{\Gamma^0} = 1 + \frac{\alpha}{\pi}\left(3\log(m_e/m_\pi) + \frac{9}{4}\right). \tag{32}$$

As was noted in [60], the same formula should apply to the QCD corrections of a pseudoscalar Higgs particle with the replacement $\alpha \to 4\alpha_s/3$. This has recently been verified in a new calculation [61]. When it comes to the structure dependent part the situation is still unclear. It was claimed in [62] that these corrections are not suppressed by the helicity factor m_e^2 and that they are singular in the soft photon limit. The latter statement was shown in [63] to violate the Low theorem. However, there is now agreement that there is no helicity suppression although none of the calculations performed [63, 64, 62, 66] seem to agree in detail on the result (in fact, an exact calculation, keeping the electron mass finite throughout, remains to be done). At any rate, it seems that the uncertainty in these radiative corrections can be avoided by having a good enough invariant mass resolution of the detector.

When it comes to η and η' decays into a lepton pair, the predictions are more model dependent. However, it has been argued [67] that by simultaneously measuring $\eta, \eta' \to e^+e^-$ and $\mu^+\mu^-$, one may extract the troublesome real parts of the electromagnetic amplitude. This is an experimental challenge, however, since the branching ratio for the e^+e^- mode is expected to be 10^{-8} or less.

As these mesons contain s quarks, and as the $\mu^+\mu^-$ channel is kinematically open, the leptonic decays may potentially be very sensitive to pseudoscalar Higgs-like particles (since those tend to couple proportionally to the fermion mass).

It has been noticed that there may exist an observable CP-violating asymmetry in the decay $\eta \to \mu^+\mu^-$, if the minimal Standard model is augmented by a scalar sector containing electroweak singlets [68]. (In most other models examined as well as in the Standard Model [69], the asymmetry is constrained by other data to be much too small to be measurable.)

So far we have discussed electromagnetic decays within the standard model, that have more or less definite theoretical predictions and that should be measurable at the new facilities like CELSIUS, Saturne at Saclay etc. We have concentrated on the decays of neutral non-strange particles, since these have been less well studied experimentally. We now turn to processes mediated by the weak interaction. Since these have to proceed through virtual W or Z^0 exchange, they will typically be suppressed by factors of 10^{-8} or more. This means that their observation will require very clean, background-free experimental conditions. The fact that weak processes are so suppressed in the standard model of course also means that putting upper limits will give important constraints on exotic contributions (for an extensive review of exotic decays of light mesons, see [70]).

In principle, the π^0 could decay into a neutrino pair if neutrinos are massive. The branching ratio is given by [71]

$$\frac{\Gamma(\pi^0 \to \nu\bar{\nu})}{\Gamma(\pi^0 \to \gamma\gamma)} \sim 3 \cdot 10^{-8} \cdot \frac{m_\nu^2}{m_\pi^2}\sqrt{1 - \frac{4m_\nu^2}{m_\pi^2}}, \tag{33}$$

which means that it is bound to be less than around 10^{-8} given the upper bound on the ν_τ mass and the fact that, according to the LEP and SLC results, there are not more than three light neutrinos.

Even smaller is the radiative decay $\pi^0 \to \nu\bar{\nu}\gamma$, which in the standard model has a branching ratio of [73]

$$\frac{\Gamma(\pi^0 \to \nu\bar{\nu}\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{G_F^2 m_\pi^4}{1920\alpha\pi^3}(1 - 4\sin^2\theta_w)^2 \sim 2 \cdot 10^{-18}, \tag{34}$$

a clearly hopelessly small value. The corresponding branching ratios for η and η' decays are around 10^{-15}.

In some non-standard models, the rates could be much larger. Recently, Giudice [74] made the interesting observation that there is a way to evade the cosmological bound [75] $m_\nu \leq 100$ eV, for the tau neutrino provided that it has a magnetic moment in the vicinity of 10^{-6} Bohr magnetons. If, in addition, it has a mass between a few MeV and 35 MeV (the present experimental upper bound) it will be non-relativistic at freeze-out and will thus constitute cold dark matter.

We write the effective photon-neutrino interaction vertex as

$$-i(\kappa\gamma^\nu k^2 + \mu\sigma^{\rho\nu}k_\rho), \tag{35}$$

where it is customary to write $\kappa = e\langle r^2\rangle/6$ (where $\sqrt{\langle r^2\rangle} \equiv$ charge radius), $\mu = \mu_0 e/2m_e$ (this gives the magnetic moment in units of Bohr magnetons).

For the Dalitz-like process one obtains [76] (the charge-radius term is negligible here)

$$\frac{1}{\Gamma_{\gamma\gamma}}\frac{d\Gamma}{dx}(\pi^0 \to \nu\bar{\nu}\gamma) = \frac{\mu_0^2\alpha m_\pi^2}{12\pi m_\nu^2 x\sqrt{x}}(1-x)^3(x + 8m_\nu^2/m_\pi^2)\sqrt{x - 4m_\nu^2/m_\pi^2}, \tag{36}$$

where $x = s/m_\pi^2$, and where the $\pi\gamma\gamma^*$ form factor has been assumed to be constant over the limited s range considered. Neglecting m_ν, one can integrate this expression over s to obtain

$$\frac{\Gamma(\pi^0 \to \nu\bar{\nu}\gamma)}{\Gamma(\pi^0 \to \gamma\gamma)} = \frac{\alpha\mu_0^2 m_\pi^2}{48\pi m_e^2}, \tag{37}$$

which means a branching ratio of $0.5 \cdot 10^{-10}$ for $\mu_0 = 4 \cdot 10^{-6}$ for π_0 decays and $3.5 \cdot 10^{-10}$ for η decays. (These results have recently been verified in another calculation [77].)

Although bigger than the standard model rates, these branching ratios seem to be somewhat too low to be detectable in the near future. As shown in [76], it is probably better to search for effects of a neutrino magnetic moment in beam dump experiments and in J/Ψ decays. There are indications from a new analysis of a previous beam dump experiment [79] that a magnetic moment large enough to provide the dark matter may be excluded.

One may note that the present experimental limit for the branching ratio for $\pi^0 \rightarrow \gamma$ + "nothing", although recently improved [78], is only of the order of a few times 10^{-4}.

The C violating decay $\pi^0 \rightarrow 3\gamma$ is expected to have an extremely small branching ratio of 10^{-31} or so [80]. The rate goes, however, as the 12th power of the meson mass raising the expectation for the η' to perhaps 10^{-19} [70]. The present experimental upper limit for the branching ratio of $\pi^0 \rightarrow 3\gamma$ is $3.1 \cdot 10^{-8}$ [81].

The P and CP violating decay $\eta \rightarrow \pi^+\pi^-$ is induced by the θ term in QCD. The upper limit on a static electric dipole moment of the neutron implies, however, that the branching ratio obtained from this term is less than around 10^{-16} [82].

A light Higgs boson could show up in several meson decay modes. A standard model Higgs particle lighter than around 60 GeV is excluded by LEP data, so the interest will have to focus on various extensions of the minimal Higgs sector. In two-doublet models there generally exists a neutral pseudoscalar Higgs. In supersymmetric models a light such Higgs is always accompanied by an even lighter scalar Higgs, so also that scenario is rule out by LEP. In the general two-doublet case there is no such relation between the masses so that Z^0 data are less restrictive (note that the pseudoscalar Higgs does not couple directly to the Z^0, so its observation at LEP is much more difficult). One possible process is $\eta \rightarrow H\gamma\gamma$, which may have a branching ratio around 10^{-9} [83], with a distinctive invariant mass distribution of the photon pair.

Recently, the radiative pion decay $\pi^\pm \rightarrow e^\pm \nu\gamma$ has attracted renewed attention (for a review containing older references, see [84]). In the standard model, this decay contains three pieces. First, there is there more or less trivial QED correction to the $\pi \rightarrow e\nu$ decay obtained by attaching a photon to the external charged particles. This is the so-called inner bremsstrahlung (IB) part. Since it is a radiative correction to the $e\nu$ decay it is proportional to the electron mass for helicity reasons. The interesting physics may reside in the so-called structure dependent (SD) parts, proportional to the vector and axial vector transition strengths. These need not be helicity suppressed and are therefore potentially important. The general differential decay distribution in the standard model is thus given by

$$\frac{1}{\Gamma_{\pi\rightarrow e\nu}} \frac{d\Gamma_{\pi\rightarrow e\nu\gamma}}{dx dy} = \frac{\alpha}{2\pi}\Big(IB(x,y) + SD^+(x,y) + SD^-(x,y)\Big), \qquad (38)$$

where $x = 2E_\gamma/m_\pi$, $y = 2E_e/m_\pi$ and where $SD^+ \sim (F_V + F_A)^2$, $SD^- \sim (F_V - F_A)^2$ are both positive definite probability densities. The ratio of F_A and F_V can be estimated in various models [84] and can also be obtained from the experimental shape of the distribution, one finds $F_A/F_V = +0.5 \pm 0.3$.

The surprising finding in a recent experiment [85] is that the best fit to the full experimental distribution is with a negative contribution from SD^-. The standard model expectation is $0.4 \cdot 10^{-8}$ for this contribution, whereas the experimentally measured value [85] is $-5.8 \pm 0.20 \cdot 10^{-8}$.

Radiative corrections have recently been shown to make the problem even worse [87]. This negative contribution from SD^- is a clearly unphysical result which calls for an explanation. It has been suggested by Poblaguev [86] that the negative value may be explained if one adds a tensor interaction of the form

$$\mathcal{L}_T = \sqrt{2} G_F V_{ud} f_T (\bar{u}_R \sigma_{\mu\nu} d_L)(\bar{e}_R \sigma_{\mu\nu} \nu_L), \tag{39}$$

which interferes destructively with the inner bremsstrahlung part. A value of $|f_T| = (4.2 \pm 1.3) \cdot 10^{-2}$ is then needed to explain the experimental result [86, 88]. In the standard model, such a tensor interaction does of course not exist at tree level but only as an induced interaction five to six orders of magnitude smaller than needed. In fact, also in reasonable extensions of the standard model, such as supersymmetric models or models with multiple Higgses, it has proven difficult to induce tensor terms of the required strength [88].

There has also appeared a convincing, but not completely watertight, argument against the existence of such a strong tensor force [89]. The point is that if this interaction exists, there is a higher order diagram with reabsorption of the photon which would contribute non-helicity-suppressed pseudoscalar part to the $\pi \to e\nu$ decay. Clearly, new improved measurements are needed to clarify this puzzling situation.

Turning finally and briefly to K decays, there have recently been reported

important new measurements by the NA31 experiment at CERN [90] and a BNL experiment [91] of the K_L Dalitz decay $K_L \to e^+e^-\gamma$, improving the world statistics of this decay by two orders of magnitude. The diagrams contributing to this decay can be divided into two classes, one where the non-leptonic weak Hamiltonian induces transitions between pseudoscalar states, and one where it acts between vector states. The latter has the interesting property of vanishing for on-shell photons, and therefore its strength can only be assessed by measuring processes like Dalitz decay or $K_L \to \mu^+\mu^-$. In [92] the relative strength of this contribution to the ordinary, vector-meson dominance type of form factor was parametrized by a parameter α. In phenomenological models for the $\Delta I = 1/2$ rule, like that of Sakurai, the value $|\alpha| = 1$ is predicted, whereas one gets $|\alpha| \sim 0.2 - 0.3$ [92] using the Shifman-Vainshtein-Zakharov [93] QCD-inspired non-leptonic Lagrangian. The value reported by NA31 [90] and the Brookhaven experiment [91], $|\alpha| = 0.27 \pm 0.10$, clearly seems to favour the SVZ approach.

An interesting application of this new experimental result is to analyze its consequences for the decay rate for $K_L \to \mu^+\mu^-$ and, in particular, for the top quark mass. An analysis along these lines was performed in [95] before there was any knowledge of α. It was realized in that work for the first time that a top quark heavier than 100 GeV might be needed to accomodate simultaneously K_L phenomenology and the long lifetime of the B mesons. The uncertainty about the value of α precluded any definite statement, however. With the new measurement, a more detailed analysis can be performed [96]. The measured value implies that the electromagnetic contribution to $K_L \to \mu^+\mu^-$ is very small (due to a cancellation of the two classes of diagrams). There are now two new measurements of $K_L \to \mu^+\mu^-$ in good agreement with each other, one from Brookhaven giving a branching ratio of $(7.0 \pm 0.5) \cdot 10^{-9}$ [97], and one from KEK giving $(7.9 \pm 0.9) \cdot 10^{-9}$ [98]. Combing these two measurements with the value of α from $K_L \to e^+e^-\gamma$, one may bound the top quark mass to be in the interval 110 GeV $< m_t <$ 170 GeV. This is a highly interesting result which is in agreement with other analyses based on completely different processes [99].

To conclude, we have seen there there are many interesting aspects of weak and electromagnetic decays of light mesons that are still to explore. With new experimental facilities soon in operation, this should continue to be an active and exciting field for many years to come.

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